Reducing 3SAT to INDEPENDENT SET

- Let F be a conjunction of n *clauses* of length 3, i.e., a disjunction of 3 propositional variables or their negation.
- Construct a graph G with $3n$ vertices that correspond to the variables in F.
- For any clause in F, connect by three edges the corresponding vertices in G.
- Connect all pairs of vertices corresponding to a variable x and its negation $\neg x$.
- F is satisfiable if and only if G contains an independent set of size n.

Solving numerical constraints

Satisfiability	over $\mathbb{Q}$	over $\mathbb{Z}$	over $\mathbb{N}$
Linear equations	polynomial	polynomial	*NP*-complete
Linear inequalities	polynomial	*NP*-complete	*NP*-complete

Satisfiability	over $\mathbb{R}$	over $\mathbb{Z}$
Linear constraints	polynomial	*NP*-complete
Nonlinear constraints	decidable	undecidable

NP-hard problems

- *Decision problem:* solution is either yes or no
- Example: Traveling salesman decision problem:
  Given a network of cities, distances, and a number B, does there exist a tour with length $\leq$ B?
- *Search problem:* find an object with required properties
- Example: Traveling salesman optimization problem:
  Given a network of cities and distances, find a shortest tour.
- Decision problem *NP*-complete $\Rightarrow$ search problem *NP*-hard
- *NP-hard problems:* at least as hard as *NP*-complete problems

Graph theoretical problems

- Shortest path *polynomial*
- Traveling salesman *NP-hard*
- Minimum spanning tree *polynomial*
- Steiner tree *NP-hard*

NP-hard problems in bioinformatics

- Multiple sequence alignment *Wang/Jiang 94*
- Protein folding *Fraenkel 93*
- Protein threading *Lathrop 94*
- Protein design *Pierce/Winfree 02*
- ...

Further complexity classes

coNP: Problems whose complement is in *NP*

PSPACE: Problems solvable in polynomial space

EXPTIME: Problems solvable in exponential time

$\vdots$

Literature

- J. E. Hopcroft and J. D. Ullman: Introduction to automata theory, languages and computation. Addison-Wesley, 1979
- M. R. Garey and D. S. Johnson: Computers and intractability. A guide to the theory of NP-completeness. Freeman, 1979
- C. H. Papadimitriou: Computational complexity. Addison-Wesley, 1994
- S. Arora and B. Barak: Computational complexity - a modern approach. Camdridge Univ. Press, 2009