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Discrete Mathematics for Bioinformatics (P1)

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Exercises 3

1. Skip lists (Niveau I)

Compute the expected value for the height (h), search time and space consumption if the probability p for each coin flip to produce a 1 is $1/3$.

2. "sparse" skip list (Niveau I)

Each node in the skip list has up to two incoming directed edges pointing to other nodes in the skip list.

- Which edges are really necessary for a search and which can be removed?
- Can you give a rough estimate for the expected number of edges that can be removed?

3. Skip lists (Niveau II) Proof that the height of a skip list has expected value $O(\log n)$ with high probability.

Hint: You do not need Chernoff bounds or Markov's inequality to show this.

4. Tail estimates (Niveau II)

Show that randomized Quicksort has expected run time $O(n \log n)$ with high probability.

Hint: Use the idea of successful/ unsuccessful splits. Let a split be successful if it splits the total set into two fractions such that both contain less than 75% of the complete set. What are the probabilities of these splits and how many are required to sort?