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Discrete Mathematics for Bioinformatics (P1)

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Exercises 2

1. **Modulo Arithmetic (Niveau II)** Prove the following theorem:
   For any positive intergers a and n, if $d = gcd(a, n)$ (the greatest common divisor of a and n), then
   $$\langle a \rangle = \langle d \rangle = \{0, d, 2d, \ldots, n - d\}$$
   and thus
   $$|\langle a \rangle| = n/d$$
   $(\langle a \rangle := \{a \cdot i \mod n \mid i \in \mathbb{N}\})$.
   Hint: Use Bezout's lemma. It states that if a and b are nonzero integers with greatest common divisor d, then there exist integers x and y such that $ax + by = d$

2. **Hashing (Niveau II)** Consider a version of the division method in which $h(k) = k \mod m$, where m=$2^p - 1$ and k is a character string interpreted in radix 2^p. Show that if string x can be derived from string y by permuting its characters, then x and y hash to the same value.

3. **Hashing (Niveau I)** Use a programming language of your choice to simulate the length of the lists for hashing with chaining using different hash functions you have seen in the lecture. Present and briefly explain the results, what can you say about the average length of the lists and the length of the longest list?

4. **Expected value (Niveau I)** Proof: Define pi = Pr (exactly i probes access occupied slots)for i = 0, 1, 2, ... (Note that for $i > n, p_i = 0$). The expected number of probes is then $\sum_{i=0}^{\infty} i \cdot p_i$. Now define q_i = Pr (at least i probes access occupied slots). Show that $\sum_{i=0}^{\infty} i \cdot p_i = \sum_{i=1}^{\infty} q_i$.