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November 30, 2014

Algorithms

WS 2014/15

Exercises 7

1. **SAT Problem (NIVEAU I)**
   The pigeon-hole SAT problem expresses the problem of finding a way to place n pigeons in $n-1$ pigeon-holes such that no hole contains more than one pigeon. Obviously, this problem is unsatisfiable.

   - Model the Pigeon-hole SAT problem. (See script: Literals, clauses, clause-sets)

2. **Turing machine simulation (NIVEAU I)**
   Given a Turing machine M accepting the language $L = \{0^n 1^n \mid n \geq 1\}$ with accepting state q_4 and the next move function δ:

δ	0	1	X	Y	#
q_0	(q_1, X, R)	–	–	(q_3, Y, R)	–
q_1	$(q_1, 0, R)$	(q_2, Y, L)	–	(q_1, Y, R)	–
q_2	$(q_2, 0, L)$	–	(q_0, X, R)	(q_2, Y, L)	–
q_3	–	–	–	(q_3, Y, R)	$(q_4, \#, R)$
q_4	–	–	–	–	–

   Simulate M on input 0011 and 001101.

3. **Decision problems (NIVEAU II)**
   Let w_i be the i-th word in $\{0, 1\}^*$ and M_n the n-th turing machine. Consider:

   - the general halting problem K: "Does Turing machine M_n halt for input w_i?" and
   - the special halting problem K' "Does Turing machine M_n halt for input w_n?"

   (a) Prove that K' is undecidable but semi-decidable.

   (b) Use reduction to prove that K is undecidable.