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# Algorithms

### WS 2014/15

#### Exercises 6

#### 1. Tree decomposition (Niveau I)

How large is the largest piece of a any tree decomposition for a graph G of n nodes if G is a clique? Prove your answer.

#### 2. Tree decomposition (Niveau II)

As discussed in the lecture the maximum-weighted independent set (WIS) problem can be solved on a graph with small tree width using its tree decomposition. Let T be the tree decomposition of such a graph G rooted at a node r. For any node  $t \in T$  let  $T_t$  denote the subtree rooted at t and let  $G_t$  denote the subgraph associated with this subtree. Further let S be a maximum-weighted independent set of  $G_t$  subject to the requirement that  $S \cap V_t = U$ ; that is  $w(S) = f_t(U)$  and let  $S_i$  denote the intersection of S with the nodes of  $G_{t_i}$ .

Prove the following theorem:

 $S_i$  is a maximum-weighted independent set of  $G_{t_i}$ , subject to the constraint that  $S_i \cap V_t = U \cap V_{t_i}$ .

Note, that  $w(S) = \sum_{s \in S} w_s$  is the total weight of all nodes in  $S \subseteq V_t$ , and  $f_t(U)$  denotes the maximum weight of an independent set S in  $G_t$ , subject to the requirement that  $S \cap V_t = U$ , where  $U \subseteq V_t$  is an independent set.

## 3. Tree decomposition (Niveau I)

Use the algorithm presented in the lecture to compute a tree decomposition of the graph below. Use w = 4:

