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Algorithms

WS 2013/14

Exercises 6

1. **Tree decomposition (Niveau I)**

   How large is the largest piece of a any tree decomposition for a graph G of n nodes if G is a clique? Prove your answer.

2. **Tree decomposition (Niveau I)**

   Prove the following theorem:
   Let $G = (V, E)$ be a graph, T be a tree decomposition of G, and (x, y) an edge in T. The deletion of (x, y) divides T into two components X and Y. Let V_x and V_y be the 'pieces' of x and y, respectively. Then deleting the set $V_x \cap V_y$ from V disconnects G into the two subgraphs $G_X - (V_x \cap V_y)$ and $G_Y - (V_x \cap V_y)$.
   (G_M for $M = X, Y$ is the subgraph of G that consists of all nodes in the 'pieces' of M.)

3. **Tree decomposition (Niveau II)**

   Prove the following theorem:
   If graph G contains a $(w + 1)$-linked set of size at least $3w$, then G has tree-width at least w.

   Suppose, by way of contradiction, that G has a $(w + 1)$-linked set X of size at least $3w$, and it also has a nonredundant TD $(T; \{V_t\})$ of width less than w. The idea of the proof is to find a piece V_i that is "centered" with respect to X, so that when some part of V_i is deleted from G, one small subset of X is separated from another.

4. **Tree decomposition (Niveau I)**

   Use the algorithm presented in the lecture to compute a tree decomposition of the graph below. Use $w = 4$: