Prof. Dr. Knut Reinert,
Dr. Yaron Goldstein,
Sandro Andreotti

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Algorithms

WS 2013/14

Exercises 5

1. **Skip lists (Niveau I)**

   Compute the expected value for the height (h), search time and space consumption if the probability p for each coin flip to produce a 1 is 1/3.

2. **"'sparse"' skip list (Niveau I)**

   Each node in the skip list has up to two incoming directed edges pointing to other nodes in the skip list.

   (a) Which edges are really necessary for a search and which can be removed?

   (b) Can you give a rough estimate for the expected number of edges that can be removed?

3. **Skip lists (Niveau II)**

   Proof that the height of a skip list has expected value $O(\log n)$ with high probability, i.e. show that the probability that the height deviates from $\log n$ by a large factor is very low.

   Hint: You do not need Chernoff bounds or Markov's inequality to show this.

4. **Expected values (Niveau I)**

   Let X and Y be random variables:

   (a) Prove that $E(X + Y) = E(X) + E(Y)$.

   (b) Assume that X and Y are independent. Prove that $E(XY) = E(X)E(Y)$

   (c) Assume that X takes values $\{0, 1, 2, \ldots\}$. Show that $E(X) = \sum_{k=1}^{\infty} Pr(X \geq k)$.