

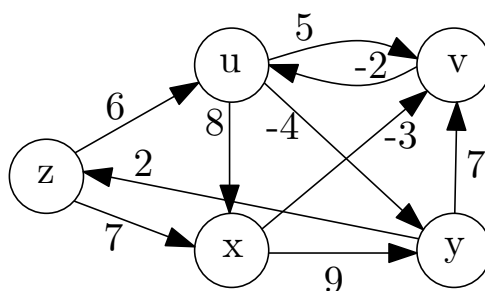
# Algorithms

WS 2013/14

## Exercises 1

### 1. Bellman-Ford (Niveau I)

Use the Bellman-Ford algorithm (see lecture script) to determine the shortest path from source  $z$  to any other node in the graph.



### 2. Bellman-Ford (Niveau II)

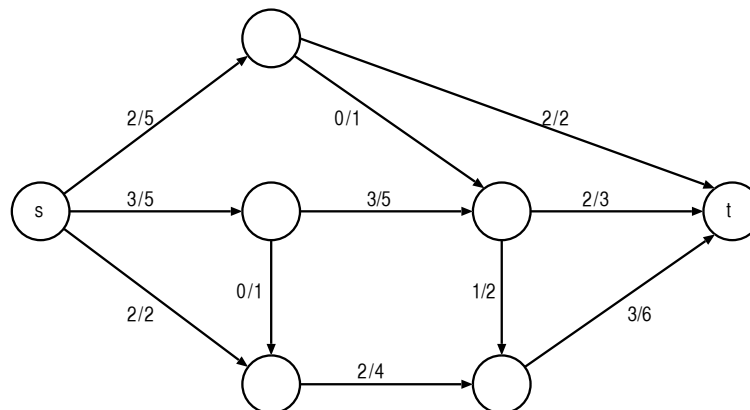
Let  $D = (V, A), n = |V|$  be a directed graph. Prove that  $D$  contains a circuit of negative length reachable from  $s$  if and only if  $f_n(v) \neq f_{n-1}(v)$ , for some  $v \in V$ , where  $f_k(v) = \min\{l(P) \mid P \text{ is an } s - v \text{ walk traversing at most } k \text{ arcs}\}$

### 3. Shortest paths in DAGs (Niveau I)

To detect the shortest path in a directed acyclic graph (DAG) allowing also for negative edge weights you could obviously apply the Bellman-Ford algorithm. Present a simpler and more efficient algorithm that exploits the acyclic property.

#### 4. Ford-Fulkerson (Niveau I)

(a) Use the Ford-Fulkerson algorithm to find a maximum flow in the network



Start with the initial flow  $f$ . An edge label  $f/c$  means initial flow  $f$  and capacity  $c$ .

(b) Find a minimum cut proving the maximality of the flow.