Probability and Evidence on Non-Standard Logics

Juliana Bueno-Soler
School of Technology and CLE- Centre for Logic
University of Campinas- Unicamp
Limeira, SP, Brazil

Micheal Dunn explains information as "what is left from knowledge when you subtract, justification, truth, belief, and any other ingredients" [3]. This idea of information as a pure semantic content that may be false and does not depend on the belief of an agent, fits the 'standard account' of information. Although this characterization of information differs from other claims that information has to be trustful, as in Floridi [4], it opens an approach on the characterization of data as distinctions, or lack of uniformity. We consider here that evidence is evidence for some proposition A, as in [2] and [5]. Evidence may appear in both linguistic and non- linguistic form, i.e. it may be conveyed by propositions as well as blood spots, details in a photograph, fossil records, fingerprints on a gun, documents, etc. These 'pieces of evidence' are justifications which may be fallible, partial, wrong, or even, in the happy case, conclusive. So, we define 'evidence for α' as a pair <Θ, α,> where Θ contains whatever is taken as evidence (or better, justification) for α. An important point about the relation between Θ and α is that it is not to be thought of as a relation of logical consequence. From the perspective of the formal system, the relation between Θ and α is considered as primitive, because it is external to the formal system.

So, the proposal of a logic of evidence is to model reasoning with evidence, not to explain what constitutes evidence for a proposition. In this sense, a logic of evidence should be paraconsistent, because it may involve contradictions; it may also be paracomplete, because evidence could be missing. It would also benefit from some non-standard probabiities, in the sense of [1].

FDE is the well-known and widely studied 'useful four-valued logic' proposed by Belnap and Dunn as the underlying logic of a computer, capable of dealing with information received from different sources that are not entirely reliable. LET_F is a logic of evidence and truth that extends FDE by introducing two new operators, a non-classicality operator ●, and a dual classicality operator ○. Contradictions are explained as conflicting evidence, where evidence is taken as a notion weaker than truth.

When ●α holds, four scenarios of non-conclusive evidence can be expressed:
1. Only evidence that α is true: α holds, ¬α does not hold.
2. Only evidence that α is false: ¬α holds, α does not hold.
3. No evidence at all: both α and ¬α fail to hold.
4. Conflicting evidence: both α and ¬α hold.

On the other hand, when ○α holds, two scenarios can be expressed, coinciding with classical truth and falsity:
5. Conclusive evidence for α: α is true (○α ∧ α holds).
6. Conclusive evidence for ¬α: ¬α is true (○α ∧ ¬α holds).

We also propose a probabilistic semantics for LET_F where statements P(α) and P(○α) express, respectively, the amount of evidence available for α and the degree to which the evidence for α is expected to behave classically (or non-classically in the case of P(●α)). A probabilistic scenario is paracomplete when $P(\alpha)+P(\neg\alpha)<1$, and paraconsistent when $P(\alpha)+P(\neg\alpha)>1$, and in both cases, $P(\circ\alpha)<1$. If $P(\circ\alpha)=1$, or $P(\bullet\alpha)=0$, classical probability is recovered for α.

In this way, LET_F is able to measure not only the degree of evidence of proposition α but also the degree to which this evidence for α is behaving classically, helping to understand the nature of evidence, from lacking to conflicting.

References

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