A Truthmaker Semantics Approach to Modal Logic

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Abstract

Truthmaker Semantics (TS) is a novel formal semantic framework developed in a series of articles by Kit Fine (see [1] and [2]) and inspired by the work of Bas Van Fraassen in [3]. TS is based on the notion of *state space* which is defined as a tuple $\langle S, \sqsubseteq \rangle$ where S is a non-empty set of states and $\sqsubseteq$ is a *partial order* over S. Moreover, we impose that $\langle S, \sqsubseteq \rangle$ is complete, namely every $T \subseteq S$ has a *least upper bound* $\bigsqcup T \in S$. We may indicate $\bigsqcup T$ as the *fusion* of the elements of T.

We remain neutral about the nature of $\sqsubseteq$ and the elements of S; however, in order to have a better grasp of the framework, the states in S may be intuitively understood as *objects* in the world (e. g. individuals, states of affairs etc.) and $\sqsubseteq$ as a *parthood* relation among those objects.

Given a language $\mathcal{L}$ with propositional letters $p, q, r...$ and Boolean connectives $\neg, \wedge, \vee$ where formulas are defined in the standard way as $A := p \mid \neg B \mid B \wedge C \mid B \vee C$, we can define a *state model* as a tuple $\langle S, \sqsubseteq, |.|^+, |.|^- \rangle$ consisting of a state space $\langle S, \sqsubseteq \rangle$ and valuation functions $|.|^+, |.|^- : \mathcal{L}_{prop} \rightarrow \mathcal{P}(S)$ mapping every propositional letter in $\mathcal{L}$ to a subset of S. $|p|^+ \subseteq S$ ($|p|^- \subseteq S$) should be interpreted as the set of *exact truthmakers* (*exact falsemakers*) of p, namely those states which are responsible and wholly relevant for the truth (falsity) of p (see [2]). For instance, consider the proposition B: "it is raining in Amsterdam"; we would say that B is made true by the fact that it is raining in Amsterdam as well as it is verified by the more complex fact that it is raining and windy in Amsterdam. However, we say that the former fact is an *exact* truthmaker of B, whereas the latter is not since it contains something, namely the sub-fact that it is windy in Amsterdam, which is not relevant for the truth of B. $s \Vdash A$ ($s \dashv\mid A$) indicates that s is an exact truthmaker (exact falsemaker) of the sentence A. Given a state model $\langle S, \sqsubseteq, |.|^+, |.|^- \rangle$, Kit Fine in [2] inductively defines the conditions for a state $s \in S$ to be an exact truthmaker ($\Vdash$) and falsemaker ($\dashv\mid$) of a sentence in $\mathcal{L}$. We say that a state s is as an *inexact truthmaker* of a sentence A ($s \Vvdash A$) iff there is a $t \sqsubseteq s$ such that $t \Vdash A$; for any formula A, B, we say that B is an *inexact consequence* of A ($A \Vvdash B$) iff for any state model $\mathcal{M} = \langle S, \sqsubseteq, |.|^+, |.|^- \rangle$ and any state $s \in S$, if $s \Vvdash A$ then $s \Vvdash B$.

The aim of my work is to investigate an extension of the above framework to the modal case originally presented by Johannes Korbmacher in some unpublished work (see [4]). In particular, I will systematically present and discuss Korbmacher's framework and prove some of its properties and then I will discuss an application of this framework to a modal extension of Angell's Logic (see [5]).

Korbmacher's idea for a truthmaker semantics for modal sentences of the form "Necessarily A" ($\Box A$) and "Possibly A" ($\Diamond A$) is based upon relativizing the relation of truthmaking and falsemaking with respect to possible worlds ($\Vdash_w, \dashv\mid_w$). Hence, he ends up with a new model construction called *E-Kripke model*. Given a language $\mathcal{L}_\Box$ obtained from $\mathcal{L}$ by adding modal operators $\Box$ and $\Diamond$ where modal formulas are defined in the standard way as $A := p \mid \neg B \mid B \wedge C \mid B \vee C \mid \Box B \mid \Diamond B$, an E-Kripke model is a tuple $\langle \mathcal{S}, \sqsubseteq, W, R, v^+, v^- \rangle$, where $\langle S, \sqsubseteq \rangle$ is a state space and $\langle W, R \rangle$ W is a Kripke frame and

- $v^+, v^- : W \times \mathcal{L}_{prop} \rightarrow \mathcal{P}(S)$ are valuations mapping every pair made of a world and a propositional letter to a subset of S.

$v^+_w(p) \subseteq S$ ($v^-_w(p) \subseteq S$) must be interpreted as the set of exact truthmakers (falsemakers) of p at the world w; given an E-Kripke model, Korbmacher provides the following truthmaker conditions for modal sentences:

$$\begin{array}{lcl} s \Vdash_w \Box A & \Leftrightarrow & \text{there is a function } f : W \rightarrow S \text{ and for any } v \text{ such that } wRv,\ f(v) \Vdash_v A \\ & & \text{and } s = \bigsqcup(\{f(v) : vRw\}) \\ s \dashv\mid_w \Box A & \Leftrightarrow & \text{for some } v \text{ such that } wRv, s \dashv\mid_v A \end{array}$$

the remaining semantic clauses as well as the definition of modal inexact truthmaking and modal inexact consequence are analogous to the non-modal case with the only difference that the truthmaking ($\Vdash$) and falsemaking ($\dashv\mid$) relations are relativized with respect to a possible worlds.

The above truthmaker (and falsemaker) conditions for $\Box A$ reflect Van Fraassen's intuition in [3] that "The facts that make *Necessarily A* true would then be the conjunctions of facts that

make A true in the various possible worlds" (p. 487 in [3]). Hence, an exact truthmaker s of $\Box A$ at w, analogously to the case of a conjunction, is the fusion of an exact truthmaker of A at w_1 *and* an exact truthmaker of A at w_2 and so on for all the accessible worlds w_1, w_2... from w. Conversely, an exact falsemaker s of $\Box A$ at w, analogously to the case of a disjunction, is an exact falsemaker of A at w_1 *or* an exact falsemaker of A at w_2 *or...* for some accessible world w_i from w. Korbmacher's semantic framework allows us to prove some interesting characterizations, in particular the definition of *modal inexact consequence* ($\Vdash_K$) can be characterized as logical consequence in the modal logic of First-Degree Entailment, K_{FDE}, (see [6]), namely $A \Vdash_K B$ iff $A \models_{K_{FDE}} B$.

I, then, discuss a possible application of Korbmacher's semantics. In particular, I try to extend to the modal case the notions of partial truth and preservation of valance (see [2]) and I try to provide a semantics for a modal extension of Angell's logic of Analytic Containment (see [5]). Angell's logic is aimed at accounting for a notion of entailment interpreted as analytic entailment in terms of containment of meaning, that is, a formula A entails a formula B if and only if the meaning of B is contained in that A. $A > B$ indicates that A analytically entails B. Korbmacher has shown that his framework allows us to construct a semantics for formulas of the form $A > B$ where $>$ is interpreted as analytic entailment and A and B are modal formulas. I investigate further the application of Korbmacher's framework to a modal extension of Angell's Logic, call it $AC_\Box$, in particular:

1. I show how Korbmacher's framework allows us to characterize a modal extension of the notion of partial truth introduced by Humberstone in [7] and developed by Fine in [1]. I also define a modal extension of the notion of preservation of valence introduced by Fine in [1] and show that preservation of modal partial truth can be syntactically characterize as preservation of modal valence;
2. I show how a truthmaker semantics for $AC_\Box$ forces us to accept the validity of the seriality principle $\Box A > \Diamond A$. More specifically, if we want the principle of simplification $(A \wedge B) > B$ (which is a theorem in Angell's logic) to be valid in the truthmaker semantics for $AC_\Box$ too, we need to impose a seriality constraint on the E-Kripke models for $AC_\Box$. But this constraint makes the principle $\Box A > \Diamond A$ valid. Namely, the modal extension of Angell's logic based on Korbmacher's truthmaker semantics, $AC_\Box$, is committed to the principle that the meaning of "Possibly A" is contained in the meaning of "Necessarily A";
3. I show that the first degree fragment of the logic $AC_\Box$ arising from Korbmacher's semantics can be characterized in terms of D_{FDE} (that is K_{FDE} plus seriality) consequence and preservation of valence. In particular, I outline a strategy to prove that $A > B$ is valid in Korbamcher's semantics for $AC_\Box$ if and only if $A \models_{D_{FDE}} B$ and A preserves the modal valence (or equivalently the modal partial truth) of B.

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