

Occurrences and Substitutions

The philosophical notion of an occurrence is usually considered in the context of the distinction between types and tokens.¹ In order to illustrate this notion, we investigate the following sentence taken from the poem “Sacred Emily” by Getrude Stein:

Rose is a rose is a rose is a rose.

The quoted sentence consists of three different words, but of ten word occurrences; in particular, the word “rose” occurs four times in that sentence. The latter means that there are four distinct occurrences of “rose” in the quoted sentence.² Analysing the example, we recognise that an occurrence is determined by the following three aspects:

1. *context*: An occurrence occurs *in* a syntactic entity (in the quoted sentence).
2. *shape*: An occurrence is an occurrence *of* a syntactic entity (of the word “rose”).
3. *position*: An occurrence is an occurrence *at* a specific *position*.

Occurrences in the same context of the same shape can be distinguished by their position in the context.

In the philosophical debate, occurrences are often misunderstood as tokens in contrast to types. Nevertheless, the concept of occurrences is independent of the distinction between types and tokens. Understanding the quoted sentence as a type (as the unique abstract sentence) or understanding the quoted sentence as a token (the concrete sentence on this page), in both cases the situation remains unchanged: there are four occurrences of the word “rose”, once understood as a word type and once understood as a word token.

In our talk, we provide a survey of an elaborate formal theory of occurrences and substitutions (which are a generalisation of occurrences) in formal languages. In such languages, context and shape are meaningful combinations of the well-known syntactic entities, as terms, formulae or derivations. We focus on a paradigmatic example of occurrences, namely on occurrences of terms in terms of a formal first order language.

¹Cf. Wetzel [2] for a brief survey of the treatment of occurrences in the philosophical debate about this distinction.

²Counting these occurrences is a simple task; representing such occurrences adequately is the non-trivial topic of our talk.

The crucial aspect of an occurrence is the representation of informally given positions. For this purpose, we suggest the use of *nominal forms* as introduced by Schütte [1]. These nominal forms are a generalisation of the standard syntactic entities permitting so called *nominal symbols* $*_k$ as an additional atomic expression. Roughly spoken, in the case of standard terms: if we replace an (informally given) occurrence of a subterm in a term by such a nominal symbol, then the resulting nominal form represents the position of the replaced subterm.

Due to the use of nominal terms for the representation of the positions: we are not only able to represent single occurrences (as discussed in the introductory example), but also some canonical generalisations:

1. *standard occurrences*: arbitrary many single occurrences of a given shape in a given context are represented together.
2. *multi-shape occurrences*: the represented occurrences may have, additionally, arbitrary many shapes.
3. *substitution*: extending a multi-shape occurrence by a sequence of arguments intended to replace the shapes of that multi-shape occurrence together with the result of such a replacement is a suitable formal representation of a substitution (understood as the process of such a replacement).

In contrast to the standard occurrences, the multi-shape occurrences and the (formal) substitutions are not equipped with a trivial identity relation; we have to introduce suitable normal forms representing the *same* informally given occurrences and substitutions, respectively.

In our talk, we do not only provide the formal details necessary to understand our theory of occurrences. Additionally, we intend to discuss briefly some problems only solvable on the base of such a formal theory (these are problems not solvable on the base of the inductive structure of context and shape) and to illustrate the capacity of this theory to represent adequately various informal concept known from everyday mathematics.

References

- [1] Kurt Schütte, *Proof Theory*. Springer-Verlag Berlin, 1977.
- [2] Linda Wetzel, *What are occurrences of expressions?*. Journal of Philosophical Logic, 22(2):215–219, 1993.