Husserl and Carnap on completeness

PhDs in Logic XII

In logic, completeness means both "maximality" and "sufficiency". On the one hand, it is the property of the theories which are so strong that they prove or disprove every sentence of its language. On the other hand, it is a property of the logics whose relation of provability coincides with that of logical consequence. At the beginning of the 1930s, these properties were clearly delimited by Gödel and Tarski, but a few years ago the meaning of "completeness" was rather ambiguous.

Fraenkel (1928) and Carnap (2000) attempted to solve this ambiguity and distinguished between three notions of completeness: categoricity, non forkability and decidability. In this sense, stand out a manuscript by Carnap, unpublished until 2000, where he claimed to have proved the equivalence of these three notions. This manuscript is essential to understand the history of the concept (or concepts) of completeness. Recently, papers by Da Silva (2016), Hartimo (2018) or Centrone (2010), which follow those written by Hill (1995) and Majer (1997), defend the role of Husserl in the development of this history. They argue that the notions of "relative" and "absolutely definite theory" –which are introduced to face the problems associated to the extension of our number systems- can be interpreted as "complete theory" (Da Silva, 2016) or as "categorical theory" (Hartimo, 2018).

However, their reading of Husserl is based on a series of lectures given in 1901 in Göttingen, known today as *Doppelvortrag*. In 1901, completeness was primary a property of the models, not of the theories (in particular, of the ordered field of real numbers). Furthermore, it was ten years before the publication of Whitehead & Russell (1910), so his notion of "inference" was purely informal. Therefore, from an historical point of view, the claims of Da Silva, Hartimo and Centrone are maybe too strong.

I claim that the intuitions underlying the ideas of categoricity, non forkability and decidability are also behind the concept of an "absolutely definite theory", i.e. an absolutely definite theory has an unique model, does not admit independent propositions and decides the truth or falsity of every proposition of its language. Thus, Husserl (2003) was informally anticipating these three notions of completeness. My argument rests on textual evidence from the *Doppelvortrag*, and also on a comparison with the work of Veblen (1904). In addition to this, I show that both Fraenkel (1928) and Carnap (2000) quoted Husserl while explaining the notion of "decidability" ("*entscheidungsdenitheit*").

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