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2-calibration on 3-manifolds and Dehn twists

Abstract: A closed 2-form of rank two on a 3-manifold is called 2-calibration of the 3-manifold. This is equivalent to having a non-singular divergence free vector field on the 3-manifold. Typical examples are contact 3-manifolds with a fixed contact form α . Then $d\alpha$ is the calibration.

David Martinez Torres studied codimension 1 foliations on $(2n+1)$ -dimensional manifolds with 2-calibrations, namely, a closed 2-form of rank $2n$ which restricts to a symplectic structure on each leaf. If we find a Lagrangian n -sphere L on a leaf, we can consider the generalized Dehn twist along L which produces a new calibrated foliation. It is also interpreted as a generalized Dehn twist of the manifold and also interpreted as a surgery described by attaching a symplectic handle of dimension $2n + 2$. He also developed a Donaldson theory (approximate holomorphic geometry) to analyze the dynamical structure of 2-calibrated foliations.

This talk is an introduction to the surgery part in his paper restricted to dimension 3. As a corollary we show that on most closed oriented 3-manifolds any 2nd real cohomology class is represented by a 2-calibration.