Leafwise symplectic structures on Lawson's foliation on S^5

Historical remark (Elmar): Up to 1970 despite many efforts, the only sphere known to admit a codimension 1 foliation was the 3-sphere due to an example of G. Reeb from the 1940's. This situation changed drastically when H. Blaine Lawson used the work of J. Milnor on topology of isolated hypersurface singularities to produce a codimension 1 foliation on the 5-sphere and on all spheres of dimension $2^k + 3$. The foliation on the 5-sphere can be very explicitly described and is quite beautiful. Efforts to put extra structures like complex or symplectic structures on the leaves of this or related foliations were successful only recently.

Abstract: We show that Lawson's foliation on the 5-sphere admits a smooth leafwise symplectic structure. The main part of the construction is to show that the Fermat type cubic surface admits an end-periodic symplectic structure. This is related to a so called $\tilde{E}_6$ simple elliptic singularity. Almost the same construction works also for $\tilde{E}_7$ and $\tilde{E}_8$, which are, together with the Fermat cubic $\tilde{E}_6$, known to be the only isolated hypersurface simple elliptic singularities. On the other hand, it seems to be a very rare case that a Stein surface (or even higher dimensional Stein manifold) admits a non-convex end-periodic symplectic structure. The recent solution of Taubes' conjecture by Friedl and Vidussi explains to some extent this impossibility. The main theorem can be paraphrased that the 5-sphere admits a regular Poisson structure of symplectic dimension 4.