Equivariant methods in Geometric Combinatorics - An Overview Lecture 3

The third lecture is an "exercise" in application of equivariant cohomology with integer coefficients in Geometric Combinatorics. The following theorem will be proved.

Theorem 1. Let $f: S^2 \to \mathbb{R}^3$ an injective continuous map. Then its image contains vertices of a tetrahedron that has the symmetry group D_8 of a square. That is, there are four distinct points ξ_1 , ξ_2 , ξ_3 and ξ_4 on S^2 such that

$$d(f(\xi_1), f(\xi_2)) = d(f(\xi_2), f(\xi_3)) = d(f(\xi_3), f(\xi_4)) = d(f(\xi_4), f(\xi_1))$$

and

$$d(f(\xi_1), f(\xi_3)) = d(f(\xi_2), f(\xi_4)).$$

Details can be found in the paper "Tetrahedra on Deformed and Integral Group Cohomology" by G. Ziegler and P. Blagojević:

http://www.combinatorics.org/Volume_16/PDF/v16i2r16.pdf.