## Equivariant methods in Geometric Combinatorics - An Overview

## Lecture 3

The third lecture is an "exercise" in application of equivariant cohomology with integer coefficients in Geometric Combinatorics. The following theorem will be proved.

Theorem 1. Let $f: S^{2} \rightarrow \mathbb{R}^{3}$ an injective continuous map. Then its image contains vertices of a tetrahedron that has the symmetry group $D_{8}$ of a square. That is, there are four distinct points $\xi_{1}, \xi_{2}, \xi_{3}$ and $\xi_{4}$ on $S^{2}$ such that

$$
d\left(f\left(\xi_{1}\right), f\left(\xi_{2}\right)\right)=d\left(f\left(\xi_{2}\right), f\left(\xi_{3}\right)\right)=d\left(f\left(\xi_{3}\right), f\left(\xi_{4}\right)\right)=d\left(f\left(\xi_{4}\right), f\left(\xi_{1}\right)\right)
$$

and

$$
d\left(f\left(\xi_{1}\right), f\left(\xi_{3}\right)\right)=d\left(f\left(\xi_{2}\right), f\left(\xi_{4}\right)\right)
$$

Details can be found in the paper "Tetrahedra on Deformed and Integral Group Cohomology" by G. Ziegler and P. Blagojević: http://www.combinatorics.org/Volume_16/PDF/v16i2r16.pdf.

