Classification of inscribable stacked polytopes

Background:

A polytope is the convex hull of a set of points.

A polytope can be seen as a cell complex of different dimensional objects, called faces, like vertices, edges, and so on. The f-vector of a polytope simply counts the number of k-dimensional faces. Two polytopes are called to have the same combinatorial type, if they have the same abstract cell complex (without coordinates). For example all planar 5-gons are of the same type.

A polytope is called simplicial, if its (d-1) dimensional faces are simplices.

Example: The octahedron and icosahedron are simplicial, cube and dodecahedron are not.

A polytope type is called inscribable, if it has a polytope that has all vertices on a sphere.

Example: A parallelepiped has the same combinatorial type as the cube and hence is of inscribable type.

The famous Lower Bound Theorem for simplicial polytopes, provides exact lower bounds for the number of k-dimensional faces in terms of the number of vertices. In dimension 4 and higher, this lower bound is exactly realized by stacked polytopes.

About the talk:

Our main result is a classification of all stacked polytopes that are inscribable. The proof is constructive. As a corollary, we get that the Lower Bound Theorem is also tight for inscribable polytopes.