# From Kneser graphs to Hom complexes

**Abstract:** In 1978 Lovász answered a combinatorial question posed by Kneser in 1955 and determined the chromatic number of graphs which we now know as Kneser graphs. A surprising feature of this proof was that it associated to each graph a simplicial complex and related the chromatic number of the graph to topological properties of this complex. This inspired a shorter and more direct proof by Bárány (1987), which rests on the same topological result as Lovász', the Borsuk-Ulam Theorem, but bypasses complexes.

I will present these two proofs and show how they are related. This will also give me an opportunity to present Hom complexes, which were introduced later to study the graphs homomorphisms by topological means and provide an enrichment of the category of graphs. Indeed, my presentation of Bárány's proof and its connections to Lovász' will be anachronistic and use later ideas by Schrijver, Ziegler, Björner, de Longueville, Babson, Kozlov, Dochtermann and others.