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Chromatic localisations of algebraic K-theory

Abstract: I will report on joint work with L. Meier and G. Tamme on an extension of a classical result of Waldhausen in rational algebraic K-theory to higher chromatic heights. I will begin by reviewing the notion of truncating invariants and their properties following my earlier work with Tamme. By a result of Bhatt-Clausen-Mathew, $K(1)$-local K-theory is nilinvariant for discrete rings, and one can deduce that $K(1)$-local K-theory is truncating on HZ-algebras. I will then explain the ingredients to our proof that $K(1)$-local K-theory is truncating on $K(1)$-acyclic algebras which admits a natural analog in higher chromatic heights. I will finish with some applications and open questions.

October 28^{th}, 2020

Hermitian K-theory for stable infinity-categories

Abstract: I will report on recent joint work with Calmès, Dotto, Harpaz, Hebestreit, Moi, Nardin, Nikolaus, and Steimle. The goal of the talk is to give an overview of the setup we develop to deal with hermitian K-theory, that of Poincaré infinity categories. This setup has similarities with the work of Blumberg-Gepner-Tabuada on universal characterisations of algebraic K-theory, (there will be a general notion of localising invariants) but also has new features not present in the context of ordinary stable infinity categories (there is a notion of bordism invariant localising invariants). The main structural result is that Grothendieck—Witt theory is the universal additive functor, and algebraic L-theory is its bordism invariant approximation. This leads to a general fibre sequence relating GW-theory to L-theory and the C_2-orbits on K-theory.

Our general setup is not only internal, indeed, for a ring R and a chosen type of forms on projective modules over R (such as symplectic, quadratic, symmetric...) we can associate a canonical Poincaré infinity-category whose GW-theory is the classical group completion of the groupoid of the chosen type of forms; this is a result of Hebestreit—Steimle. As an immediate consequence we obtain periodicity phenomena in hermitian K-theory of rings in which 2 is not invertible (this was really the open case) in parts conjectured by Karoubi and Giffen.

In addition, for these Poincaré categories, we can use Ranicki's algebraic surgery to describe the associated L-theory. In the case of Dedekind rings, these L-theories can be understood very well, leading to a resolution of the homotopy limit problem for rings of integers in number fields, finite generation of GW-groups for these rings, and the (to me) surprising result that for Noetherian rings of finite global dimension, the comparison map from quadratic to symmetric GW-theory is an isomorphism in a high range, and both exhibit a reminiscence of classical Karoubi periodicity in the common high range.