Freie Universität Berlin
Institut für Mathematik

Prof. Dr. Elmar Vogt
Sebastian Meinert

Free Groups and Graphs

Winter 2012/2013

Homework 8 — Due: December 10, 2012

Problem 1

Let F_n be a finitely generated free group and $H \subseteq F_n$ a subgroup of finite index j. Determine the rank of H.

Problem 2

Let $f\colon (\tilde{\Gamma}, \tilde{x}) \to (\Gamma, x)$ be a covering of connected graphs. A *deck transformation* is a graph automorphism $\phi\colon \tilde{\Gamma} \xrightarrow{\cong} \tilde{\Gamma}$ such that $f \circ \phi = f$. The covering f is called *normal* if for each vertex $v \in V(\Gamma)$ and each pair of lifts $\tilde{v}, \tilde{v}' \in f^{-1}(v)$ there exists a deck transformation $\phi\colon \tilde{\Gamma} \xrightarrow{\cong} \tilde{\Gamma}$ such that $\phi(\tilde{v}) = \tilde{v}'$.

Show that the covering f is normal if and only if $f_*(\pi_1(\tilde{\Gamma}, \tilde{x})) \leq \pi_1(\Gamma, x)$ is a normal subgroup.

Problem 3

Let $N \trianglelefteq F_n$ be a nontrivial normal subgroup. Show that N is either of finite index in F_n or not finitely generated.