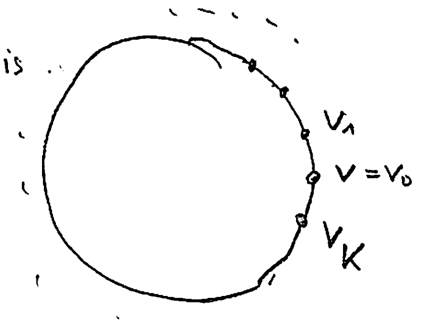
A particularly convincing example, where our algorithm gives a much smaller immersion than the covering construction is (if we ignore the trivial subgp. $H = \{1\}$ to which corresponds the universal covering, an infinite graph unless $\pi_1(\Delta, w) = \{1\}$) is the subgroup H of $\pi_1(a \infty b, w)$ generated by any single loop $p = a^{n_1} b^{m_1} \cdots a^{n_k} b^{m_k}$.

Then Γ is



$$K = \sum_i (|n_i| + |m_i|)$$

if we assume that all $n_2, \dots, n_k$ and $m_1, \dots, m_{k-1}$ are different from 0.

The corresponding covering is always infinite.

The next Proposition has a theorem of Howson from 1954 (A.6.) as a corollary. It is in a sense the dual of 3.16

<u>3.21 Proposition.</u> (Pullback of immersions represents intersection)

Let

$$\begin{array}{ccc} \Theta & \xrightarrow{g_1} & \Gamma_1 \\ {\scriptstyle g_2}\downarrow & & \downarrow{\scriptstyle f_1} \\ \Gamma_2 & \xrightarrow[f_2]{} & \Delta \end{array}$$

be a pullback diagram of graphs, and assume that f_1 and f_2 are immersions, let v_1, v_2, w be vertices of Γ_1, Γ_2 and Δ s.t. $f_i(v_i) = w$, and let v be the corresponding vertex of Θ. Let $f_0 = f_1 \circ g_1$ $(= f_2 \circ g_2) : (\Theta, v) \to (\Delta, w)$. Then

$$f_0(\pi_1(\Theta, v)) = f_1(\pi_1(\Gamma_1, v_1)) \cap f_2(\pi_1(\Gamma_2, v_2))$$

(in $\pi_1(\Delta, w)$).

Pf. „$\subset$" is obvious. So let p_1 and p_2 be reduced loops in Γ_1 and Γ_2 based at v_1 and v_2 s.t. $f_1(p_1) \simeq f_2(p_2)$.

By 3.17a $f_1 p_1$ and $f_2 p_2$ are reduced, and by 2.14 they are equal. Then the pull back property gives us a path p_0 in Θ, $\underbrace{\text{s.t.}}_{\text{based at } v_0,}$ $p_i = g_i p_0$, $i = 1, 2$ and this path is a loop. Then $f_0[p_0] = [f_1(p_1)] = [f_2(p_2)]$. $\square$