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Homotopy Theory

Summer 2015

Homework 5 — Due: May 21, 2015

Problem 9

Show that a Serre fibration has the Homotopy Lifting Property for every CW-complex.

Hint: Look at Hatcher and work out the mssing details.

Problem 10

Let $p : E \to B$ be a fibration and ω a path in B from x_0 to x_1. Look at

$$\begin{array}{ccc} p^{-1}(x_0) \times \{0\} & \hookrightarrow & E \\ \downarrow & & \downarrow \\ p^{-1}(x_0) \times I & \longrightarrow & B \end{array}$$

where the bottom map is given by $(y, t) \mapsto \omega(t)$.
Show that there is an associated map

$$h_\omega : p^{-1}(x_0) \to p^{-1}(x_1).$$

such that

(i) For homotopic paths ω, ω' from x_0 to x_1, there is a homotopy $h_\omega \simeq h_{\omega'}$.

(ii) The map h_ω is a homotopy equivalence for every path ω.