Freie Universität Berlin
Institut für Mathematik

Elmar Vogt
Filipp Levikov

Homotopy Theory

Summer 2015

Homework 12 Due: July 9, 2015

Let $f : X \to Y$ be a pointed map. The homotopy cofibre hocof(f) of f is by definition the reduced mapping cone of f and the sequence

$$X \to Y \to \text{hocof}(f)$$

is called a cofibration sequence. Dually, the homotopy fibre hofib(f) of F is the fibre of the fibration

$$P_f \to Y, \quad (x, \omega) \mapsto \omega(0)$$

over the base point of Y, where $P_f = \{(x, \omega) \in X \times Y^I \mid \omega(1) = f(x)\}$. Correspondingly the sequence

$$F_f \to X \to Y$$

is called a fibration sequence.

Problem 22

(i) Define a (non-trivial) "coaction" map"

$$\mu : \text{hocof}(f) \to X \vee \text{hocof(f)}$$

such that for all pointed spaces Z, the map

$$[\Sigma X; Z]_0 \times [\text{hocof}(f); Z]_0 \cong [\Sigma X \vee \text{hocof}(f); Z]_0 \to [\text{hocof}(f); Z]_0$$

defines an action of the group$[\Sigma X; Z]_0$ on the set $[\text{hocof}(f); Z]_0$.

(ii) Show that $a, b \in [\text{hocof}(f); Z]_0$ are mapped to the same element in $[Y; Z]_0$ by the map induced by the standard inclusion of Y in hocof(f) if and only if they are in the same orbit of this action, i.e. if and only if there is $g \in [\Sigma X; Z]_0$ with $ga = b$.

(Please turn over.)

Problem 23

Show that applying the functor $\mathrm{Map}(-, Z)$ to a cofibration sequence one obtains a fibration sequence, where $\mathrm{Map}(A, Z)$ denotes the space of base point preserving continuous maps $A \to Z$ with the compact-open topology.