Surfaces and Automorphisms

Problem Set 1 H. Reich/F. Lenhardt
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Exercise 1
Recall that $T(f) = g_{\text{fund}} \star f$, where $g_{\text{fund}} = \frac{1}{\pi z}$ and $f \in C_c^1(\mathbb{C})$, and $S = \partial \circ T$. Show that S is scale invariant, i.e. it commutes with the operator s_r that is given by

$$s_r(f) = f \circ l_r$$

where $l_r(z) = rz$ for $r \in \mathbb{R} - \{0\}$.

Exercise 2
Show that for $f \in C_c^0(\mathbb{C})$ and $\alpha \in (0,1)$, the function $T(f)$ is α-Hölder continuous. If you get stuck, check Hubbard, Theorem A.6.3.2.

Exercise 3
Show that every $f \in C^1(\mathbb{R}^n)$ is locally α-Hölder continuous.

Exercise 4
Define $\phi \colon \mathbb{R}^2 \to \mathbb{R}$ as

$$\phi(z) = -\frac{1}{2\pi}\log(\|z\|)$$

Show that ϕ is in $L^1_{loc}(\mathbb{R}^2)$ and that $\Delta\phi = 0$ on $\mathbb{R}^2 - \{0\}$, where Δ is the Laplace operator.
Remark: One can indeed show that ϕ is a fundamental solution for the Laplace operator: Given a C^2-function g, $\phi \star g$ solves $\Delta f = g$.