Surfaces and Automorphisms

Problem Set 2 H. Reich/F. Lenhardt
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Exercise 1
Let $p\colon X \to Y$ be a covering map.

(i) Similarly to the frame bundle associated to a vector bundle, show that there is a Σ_n-principal bundle associated to p.

(ii) Define a map from the set of isomorphism classes of transitive Σ_n-actions to the set of isomorphism classes of covering spaces over Y.

Exercise 2
Let V be a finite-dimensional real vector space.

(i) Define a natural map $X^{\text{inn}}(V) \to X^{\text{conf}}(V) \times X^{\text{dens}}(V)$ and show that it is an isomorphism.

(ii) Define a natural map $X^{\text{inn}}(V) \times X^{\text{Or}}(V) \to X^{\text{conf}}(V) \times X^{\text{Vol}}(V)$ and show that it is an isomorphism.

Exercise 3
We consider the space $E = \text{Emb}_{\mathbb{R}}(\mathbb{R}^2, \mathbb{R}^3)$ of $\mathbb{R}$-linear embeddings from $\mathbb{R}^2$ into $\mathbb{R}^3$. The group $Gl_2(\mathbb{R})$ acts on E by precomposition. We define the Grassmann manifold of type $(2,3)$ as

$$\text{Grass}(2,3) = E/G$$

(i) Show that $\text{Grass}(2,3) = E/G \cong \mathbb{R}P^3$.

(ii) Show that the quotient map $E \to \text{Grass}(2,3)$ is a $GL_2(\mathbb{R})$-principal bundle.

(iii) Let F be a surface together with a smooth immersion $f\colon F \to \mathbb{R}^3$. Show that f determines a map of principal bundles

$$P_{Gl_2(\mathbb{R})}(TF) \to E$$

(iv) Show that f determines a reduction of the structure group $Gl_2(\mathbb{R})$ to $O(2)$.