Freie Universität Berlin

Surfaces and Automorphisms

Problem Set 11 H. Reich/F. Lenhardt
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Exercise 1

Let $f\colon \mathbb{C} \to \mathbb{C}$ be $\mathbb{R}$-linear. We set $K(k) = \frac{1+k}{1-k}$ for $k \in [0,1)$. Show that

$$\mu(f) \leq k$$

if and only if

$$\|f\|_{op}^2 \leq K(\det(f)).$$

Exercise 2

Classify all 2-sheeted coverings of $\mathbb{C}P^1$ that are ramified at exactly $1, 2, 3$ or 4 points.

Exercise 3

Let η be a smooth function on $\mathbb{C}$ with support in the unit disk $\mathbb{D}$ such that $\eta \geq 0$ and $\int_{\mathbb{C}} \eta = 1$. Set

$$\eta_k(z) = k^2 \eta(kz)$$

(i) If f is continuous with compact support, the sequence $\eta_k \star f$ converges uniformly to f and

$$\operatorname{supp}(\eta_k \star f) \subset \operatorname{supp}(f) + \operatorname{supp}(\eta_k)$$

(ii) Using that smooth functions with compact support are dense in $L_1(\mathbb{C})$, show that for $f \in L_1(\mathbb{C})$ $\eta_k \star f$ converges to f in the L_1-norm.

Exercise 4

Suppose T is the standard torus obtained by glueing sides of the unit square. Consider T as a Riemann surface. Suppose A is an open annulus embedded in T.

(i) Show that A is a finite annulus.

(ii) In the case where the embedding is not homotopic to a constant map, find an estimate or the modulus for A in terms of the homotopy type of the embedding.