Configuration spaces

Problem Set 7 E. Vogt/F. Lenhardt
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Exercise 1
Recall that for a fibration $p\colon E \to B$ with fibre $F = p^{-1}(b_0)$ we have for every loop ω in B based at b_0 a well-defined homotopy class h_ω of self homotopy equivalences of F, which defines a (right) action of $\pi_1(B, b_0)$ on the homology and cohomology groups of F. Consider the fibre bundle

$$F_2(\mathbb{R}^2_{k-2}) \to \mathbb{R}^2_{k-2}$$

with fibre $\mathbb{R}^2_{k-1}$.
Show that for $k = 3$ the action of $\pi_1(\mathbb{R}^2_{k-2})$ on the homology of the fibre is trivial.

Exercise 2
Do the same for $k = 4$.

Exercise 3
Recall that S_k acts freely on $F_k(\mathbb{R}^{n+1})$. Show that the quotient space $F_2(\mathbb{R}^{n+1})/S_2$ has the homotopy type of $\mathbb{R}P^n$.