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Configuration spaces

Problem Set 4 — E. Vogt/F. Lenhardt
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Exercise 1
Given pointed spaces $(X_i, b_i), i = 1, 2$, consider the embeddings $j_i \colon X_i \to X_1 \times X_2$, $j_1(x) = (x, b_2), j_2(x) = (b_1, x)$. Let $\alpha_i \in \pi_{p_i}(X_1 \times X_2, (b_1, b_2))$ be in the image of $\pi_{p_i}(X_i, b_i)$. Show that $[\alpha_1, \alpha_2] = 0$.

Exercise 2
Let X be simply connected and $f_i \colon S^{n_i} \to X, i = 1, 2$ continuous maps. Let ϕ_i be the corresponding (unpointed) homotopy classes. Define the Whitehead product $[\phi_1, \phi_2]$ as follows: Pick $x_0 \in X$ and paths ω_i from $f_i(1)$ to x_0, where 1 is the basepoint of the respective sphere. Then $[h_{\omega_1}\phi_1, h_{\omega_2}\phi_2] \in \pi_{n_1+n_2-1}$ is defined. We define $[\phi_1, \phi_2]$ to be the unpointed homotopy class of any representative of $[h_{\omega_1}\phi_1, h_{\omega_2}\phi_2]$.

(i) Show that this is well-defined.

(ii) Show that $[\phi_1, \phi_2] = 0$ if there exists a continuous map $f \colon S^{n_1} \times S^{n_2}$ such that $f\,|_{S^{n_1}\times 1} \simeq f_1$, $f\,|_{1\times S^{n_2}} \simeq f_2$.

Exercise 3
Let for $1 \leq r, s \leq k, r \neq s$ $\alpha'_{rs} \colon S^n \to F_k(\mathbb{R}^{n+1})$ be the maps defined in the lecture. For $1 \leq t \leq k-1$ let $\tau = (t, t+1) \in S_k$ be the transposition of t and $t+1$ in the symmetric group. Also consider τ as a self-homeomorphism of $F_k(\mathbb{R}^{n+1})$. Show that $\tau\alpha'_{rs} \simeq \alpha'_{\tau(r),\tau(s)}$ if $\{r, s\}$ and $\{t, t+1\}$ have precisely one element in common.