Configuration spaces

Problem Set 1 E. Vogt/F. Lenhardt
WS 2013/14 Due:

Exercise 1
Let B, F be topological spaces. Show that the projection map $B \times F \to B$ is a fibration with fiber F.

Exercise 2
Let $p\colon E \to B$ be a fibration.

(i) Let $\omega\colon [0,1] \to B$ a path connecting the points $a = \omega(0)$ and $b = \omega(1)$. Using the lifting property of p, construct a map $f_\omega\colon F_a \to F_b$ where $F_x = p^{-1}(x)$ is the fiber over x. For this, consider the map $F_a \times [0,1] \to B$ given by $(f, t) \mapsto \omega(t)$.

(ii) Show that the homotopy class of f_ω is independent of all choices you made.

Exercise 3
We continue the previous exercise.

(i) Show that if ω, ω' are two paths between a and b which are homotopic relative endpoints, f_ω and $f_{\omega'}$ are homotopic.

(ii) Let ω be a path from a to b and ω' a path from b to c. We define a path $\omega * \omega'\colon [0,1] \to B$ from a to c via

$$(\omega * \omega')(t) = \begin{cases} \omega(2t) & \text{if } t \le \frac{1}{2} \\ \omega'(2t-1) & \text{if } t \ge \frac{1}{2} \end{cases}$$

We can now define two homotopy classes maps from F_a to F_c: On the one hand, we can form the homotopy class $f_{\omega'} \circ f_\omega$, on the other hand, we have the homotopy class of $f_{\omega * \omega'}$. Show that these two homotopy classes are equal, i.e. that $f_{\omega'} \circ f_\omega$ and $f_{\omega * \omega'}$ are homotopic.

(iii) Show that for each $a, b \in B$ in the same path component, F_a and F_b are homotopy equivalent.