Surfaces and Automorphisms

Problem Set 9 H. Reich/F. Lenhardt
WS2013/14

Exercise 1

(i) On $\mathbb{C}P^1$ resepctively $\mathbb{C}$, there is no Riemannian metric that is invariant under all automorphisms from $\mathrm{Bihol}(\mathbb{C}P^1)$ respectively $\mathrm{Bihol}(\mathbb{C})$.

(ii) Is the round metric on $\mathbb{C}P^1 = S^2$ conformal? If yes, describe it in the usual coordinates for $\mathbb{C}P^1$.

Exercise 2
Let M be a Riemann surface and ρ_M the distinguished Riemannian metric of constant curvature -1. Then we have $\mathrm{Bihol}(M) = \mathrm{Isom}(M, \rho_M)$.

Exercise 3
Let $\mathcal{G}$ be the pseudogroup on $\mathbb{C}P^1$ generated by restrictions of Möbius transformations to open subsets of $\mathbb{C}P^1$. We obtain the associated notion of a projective structure on a topological surface.

(i) Every projective structure determines an underlying Riemann surface structure.

(ii) Every Riemann surface structure can be refined to a projective structure.

(iii) There is no Möbius transformation f from $\mathbb{B} = \mathbb{R} \times (-\pi, \pi)$ onto $\mathbb{D}$.

(iv) There exists more than one projective structure on $\mathbb{D}$.

(v) Do these different structures define different points in the moduli space of projective structures on $\mathbb{D}$ modulo pullback via biholomorphic automorphisms?
A point in the moduli space of projective structures is represented by a projective structure on $\mathbb{D}$, and we identify two such points if one is obtained from the other via pullback along a biholomorphic automorphism.