Using Network Density as a New Parameter to Estimate Distance

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***Abstract*—A wireless sensor network consists of a large quantity of small, low-cost sensor nodes that are limited in terms of memory, available energy and processing capacity. Generally, these sensor nodes are distributed in space to obtain physical parameters such as temperature, humidity, vibration or light conditions, and transmit the measured values to a central entity. The measurements are tagged with the corresponding location of the nodes in the network and the time of sampling, to enable a view on the value distribution in space and time later on. Positioning of wireless sensor nodes without dedicated hardware is an open research question. Especially in the domain of embedded networked sensors, many applications rely on spatial information to relate collected data to the location of its origin. As a first step towards localization, an estimation of the distance between two nodes is often carried out to determine their positions. So far, the majority of approaches therefore explore physical properties of radio signals such as the strength of a received signal or its trip time. However, this is problematic since either the complexity on the software or on the hardware side is not adequate for embedded systems, or the approaches lack the required accuracy.**

In this paper we present the WDNI algorithm (Weighted Density of Node Intersection) to determine the distance between two nodes, relying solely on the investigation of local node densities. To evaluate the accuracy of this algorithm, we ran extensive simulations and experimented with different testbed setups using real sensor nodes, and finally compared WDNI to a range-free distance estimation algorithm based the analysis of RSSI values.

I. Introduction

The estimation of the position of every node in Wireless Sensor Networks (WSN) is still an open question. Due to the intrinsic properties of sensor nodes, it is necessary to find localization methods that work in an ad-hoc fashion and without additional specialized hardware to save scarce resources. This is especially important since it is not possible to rely on GPS in indoor scenarios. To obtain this information there is a variety of techniques that exploit physical phenomena such as the time of arrival (TOA) of sound signals [8], the time difference of arrival (TDOA) between radio and ultrasonic signals [7], the use of interferometry [5], radio signal strength indicator (RSSI) [12], or making use of camera pictures with a previous scene analysis [6]. The accuracy that these systems are able to provide comes at the cost of a high synchronization overhead, thus high energy expenses at runtime and the need for dedicated hardware on the sensor nodes. In contrast, range-free algorithms rely solely on conventional hardware of sensor nodes.

In this paper we propose a novel method to estimate distances based on views of sensor nodes on the network densitiy called Weighted Density Node Intersection (WDNI). On the one hand, we use the number of common nodes within the radio range intersection area between two nodes that share a communication link. On the other hand we also take into account the number of nodes in the union of communication ranges to weight the distance estimation. The proposed technique targets and is evaluated for indoor usage and corresponding ranges.

The structure of the remaining paper is as follows: First we introduce in section II a mathematical model to relate the number of nodes in the intersection area with the distance between them using an approximation function. Making use of the `ns-2` simulator in section III and a real testbed in Section IV, we probe the quality of WDNI in uniform and near-uniform node distributions with different node densities. The impact of different network configurations on the distance error is evaluated, and put into context of current research with the help of a quantitative analysis against a RSSI-based distance estimation in section V. Finally, in section VI we discuss other approaches for distance estimation, give an outlook on future work in section VIII and summarize our findings with the conclusion.

II. Weighted Density of Node Intersection

The basic idea of WDNI is to approximate distances between two nodes using only the knowledge about local node densities, which can be described as the number of nodes within the communication range of another. To derive a distance from this knowledge and implement an algorithm accordingly, we proceed in the following three steps:

The first step is to find a mathematical expression of the distance between two nodes in terms of the intersection area of their communication ranges. This distance we relate to the number of the neighboring nodes in the intersection area, constructing an approximation by means of evaluating different sized uniformly distributed networks. Finally, we weight this distance with the number of nodes in the union of their communication ranges.

WDNI is based on an idealized radio model to define an approximation model. Although we are aware of this assumption not being true in reality, we use it to simplify the mathematical foundation. Later on, we will verify the validity in a real testbed scenario, thus relax these strict preconditions. Three main assumptions were taken into consideration:

1) Unit disc graph radio transmission range.
2) Identical transmission ranges for all the nodes in the network.
3) Uniform distribution of nodes in the network.

The normalized intersection area A_n of two overlapping transmission ranges of sensor nodes can be obtained by solving the Circle-Circle Intersection equation [9]. A_n denotes the normalized intersection area of the circles, whereas d_n is the normalized distance between the nodes.

$$A_n = 2R^2 \cos^{-1}(\frac{d_n}{2R}) - \frac{d_n}{2}\sqrt{4R^2 - d_n^2} \tag{1}$$

Since we are interested in finding an expression for the distance, we have to solve equation 1 for d_n. We verified that the a polynomial of degree 3 is sufficient for our algorithm.

$$d_n = 0.0239A_n^3 + 0.245A_n^2 - 1.006A_n + 1.9254 \tag{2}$$

Equation 2 now gives us the normalized distance between nodes that share intersecting transmission ranges dependent on the normalized area of this intersection.

Due to the restricted resources available on real sensor nodes it is not suitable to design an algorithm that requires in-situ complex mathematical computations. The idea is to map this problem to a more light-weight one by demanding the sensor nodes to conclude their distances from the number of their neighboring nodes. Under the assumption of uniformly distributed networks, the number of nodes in the intersection area A_i is proportional to this area. We refer to this quantity of nodes as K_i.

$$A_n \sim p(d, K_i) * K_i \tag{3}$$

where $p(d, K_i)$ describes the relationship between distances between nodes and the number of nodes in the intersection area. We obtain this approximation by running a multitude of simulations with a variety of node densities.

The graphs plotted in Figure 1 have been derived using the ns-2 network simulator. Into a rectangular area of 400 x 400 meter, a number of nodes ranging from 5 to 100 nodes have been deployed in a uniform fashion. For our approximation, the radio communication range was set to 250 meter. This way, we vary K_i, getting an idea about its behavior in networks with low, medium and high density. To plot each graph, the average distance between two nodes for a given K_i using 20 simulations has been taken into account. Note that for better comparability both axis of the graph were normalized. As can be seen clearly, the different graphs of Figure 1 converge in the function $p(d, K_i)$. Therefore, we can use the knowledge about K_i to determine the distance between nodes being aware of the transmission range and the network density.

Although using the distance approximation $p(d, K_i)$ in equation 2 is valid, the error of the estimation can be further minimized taking into account the local node densities of participating nodes. The approximation can be smoothed by

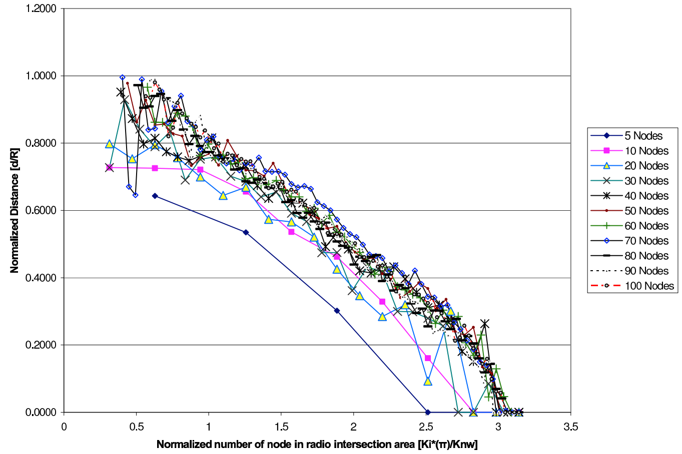

Fig. 1. Approximation graphs to estimate distance depending on K_i for different node densities.

weighting it with the number of nodes that are in the union of their transmission ranges, which we denote as K_u.

$$A_n \sim \frac{p(d, K_i) * K_i}{K_u} * \pi \tag{4}$$

The equation is multiplied by π due to circular, normalized transmission ranges, thus a unit disk graph model.

Using equation 4 in combination with equation 2, a deployed sensor node in a network can estimated its distances to adjacent nodes. Therefore the node needs to solely rely on knowledg about local node densities.

III. Simulating the WDNI Algorithm with ns-2

The first step to determine whether WDNI can be used in a network setting has been to test the algorithm with the help of the network simulator `ns-2`. A clear focus has been to study its behavior under different network settings, especially varying network densities. Simulations feature a fixed number of 100 nodes. The network size was increased until the density of nodes became too sparse, thus the network disconnected. The same effect can be obtained when changing the transmission range of the nodes accordingly. Every node in the network computes its relative distances to those nodes that are in its transmission range by using WDNI. The results of the simulations are depicted in Figure 2 for uniform and 3 for non-uniform network distributions. Here, the corresponding absolute error of the estimation is simply the absolute value of the difference of the actual distance between nodes and the calculated distance. In the following, the normalized error is therefore calculated by dividing the absolute error by the radius of the transmission range of a node.

To be able to also compare the error independent of variable radio communication ranges and network sizes, we define the Space-Range Ratio (SRR) depicted on the x-axis. This is simply the radio communication scope of the idealized node over the length of one of the sides of the deployment area. A value of 1 for SRR therefore equals a transmission range covering the complete network. We chose to use interquartile

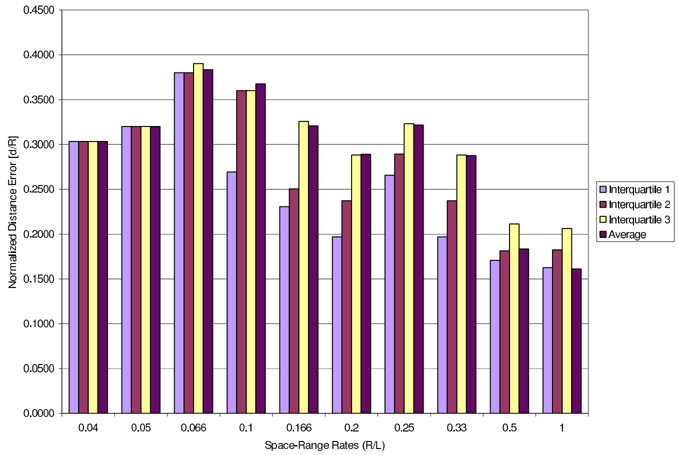

Fig. 2. Absolute normalized error in distance estimation vs. covered radio range in a simulated, uniformly distributed network .

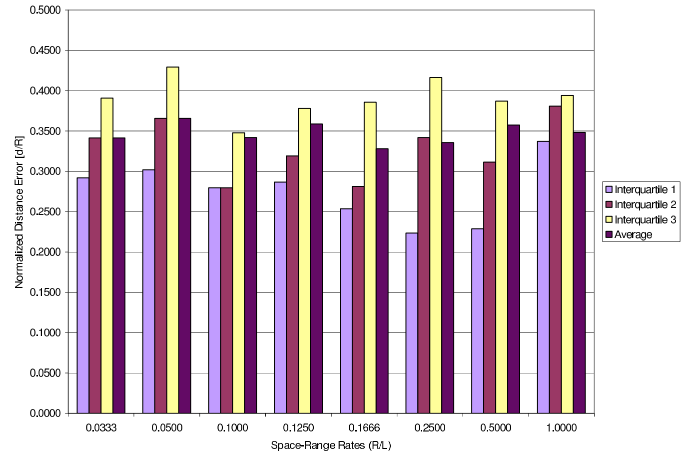

Fig. 3. Absolute normalized error in distance estimation vs. covered radio range in a simulated, near-uniformly distributed network.

diagrams since it is then possible to judge the value dispersions of the distance errors.

A. Uniform network distribution

In Figure 2, the absolute normalized errors in distance estimations of ten simulation runs are shown. Nodes that can not derive their distances to other ones, due to the lack of neighboring nodes within their range, are not considered.

As expected, the absolute normalized distance error shows the trend to decrease with increasing network connectivity. Furthermore, the error for 75% of the estimations is less than 0.4R, see interquartile 3. We can see that for the best 25% as shown in interquartile 1 with an SRR value of 1, we obtain an error of 0.16R. This is due to the fact, that nodes have more neighboring nodes than in sparse networks. A number of at least ten nodes in common with the node to derive the distance to can be used as rule of thumb to get reasonable distance estimations. While increasing the SRR value from 0.5 to 1 does not have in impact on the interquartiles, we can observe that the average error still decreases. This shows the influence of the worst 25% of the error estimations that are not considered in interquartile diagrams.

Interesting data points in Figure 2 are the results of an SRR value of 0.066R. Here, the error value is at its peak because nodes still have connectivity but with a small number of neighbours. When lowering the SRR values, connections start to break, thus fewer nodes add to the overall error.

B. Near-uniform network distribution

While WDNI has been developed under the assumption of uniformly distributed networks, we want to measure the accuracy of distance estimations also for near-uniform distributions in a second step. In this network distribution, the nodes were positioned in a way that the network takes the form of a horseshoe. This means that the center and one side were left empty, and all nodes spread out on the remaining three sides which allows for testing the response of WDNI to high densities both close and far away from a node with few nodes in a medium distance.

Looking at the average error at different SRR values, we can observe that it remains stable at around 0.35R. As we saw in the last section, with increasing SRR the number of neighboring nodes also increases and the distance estimation improves, which is once again observable in the interquartile 1 of Figure 3 up to an SRR value of 0.5R. Here, this positive effect is canceled out by the miscalculation of nodes at the corners, leading to this constant average error. Compared to the error value of the uniform distribution, the results at an SRR value of 1.0 in the near-uniform distribution are not as good. This effect occurs since all nodes are within each other's communication range, assume to be uniformly distributed, and this false assumption results naturally in high error rates. No differentiation is possible any more since the nodes lack discriminating information. From this we can conclude, that WDNI will work well as long as the transmission range is set to a value that enables most of the nodes to experience a neighborhood close to a uniform distribution but also enables nodes to have different local views on the network density.

IV. Experimental evaluation of WDNI

The results of the simulations were very promising, hence we decided to implement WDNI on the ScatterWeb Modular Sensor Boards (MSB) [2]. These nodes feature the 16-bit microcontroller MSP430F1612 from Texas Instruments equipped with 55 KB of flash memory and 5 KB RAM, and a Chipcon CC1020 transceiver using the ISM band at 869 MHz. The transceiver is able to monitor the received signal strength (RSSI) and the transmit power can be set directly in software.

A. Adaptation of the WDNI protocol for real world experiments

Due to the limited computational capabality of sensor nodes, we replace the computation of equation 4 and 2 with a density-to-distance lookup table. Depending on the number of nodes in common K_i and the local node density K_u, a node can derive its distance from a neighboring node.

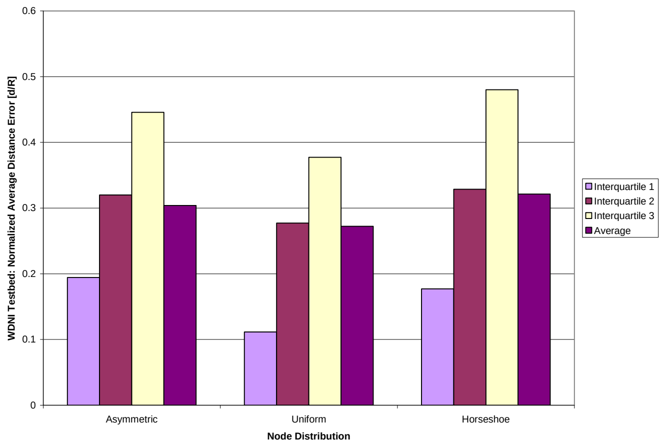

Fig. 4. Averaged normalized errors per interquartile in a asymmetrical, horseshoe and uniform distribution derived from WDNI testbed distance estimation.

The protocol of WDNI proceeds in three phases. In phase one, every node in the network broadcast a HELLO packet to discover neighboring nodes within its communication range. To avoid collisions on the medium, we implemented a delay timer depending on the node ID. The information obtained in the first phase is a neighbor table with a single entry for each discovered neighbor.

The exchange of neighbor tables is carried out in phase two. This is necessary since to compute the distance to an adjacent node, each node has to be aware of how many nodes it has in common with a neighbor, and how many nodes are in the union of their transmission ranges. A problem that appears frequently in realistic scenarios is communication link asymmetry. In this case a sensor node can receive signals of another node perfectly but communication in the opposite direction fails. This leads to a failure in the exchange process of neighbor tables. To prevent constant retransmissions of the exchange table request, the expiration of an internal timer limits the overall waiting time. For nodes that are exposed to asymmetric links, WDNI sets the distance to a maximum value.

In the last phase, nodes consult the density-to-distance lookup table using K_i and K_u, obtained in phase two as input parameters, to finally determine the distance between themselves and another node.

B. Testbed Setup and Experimental Results

Three different testbed layouts, each featuring 21 deployed nodes, have been used for our experiments. The first layout is an asymmetrical distribution of nodes, the second depicts a horseshoe setup and the last one is a uniform distribution of sensor nodes as have already been used in the `ns-2` simulations. The nodes were deployed in a 4 x 4 meter square in a seminar room at our institute. They were placed on desks and the room was cleared over desk height. The room was big enough to assure a distance of at least 2 meter between border nodes and the walls. The distance between the nodes varied among layouts and has been between 0.5m and 2m for the asymmetrical and horseshoe testbed and 1m in the uniform layout.

Since our algorithm depends on a spherical radio propagation model described in section II, we depend on mapping a transmission power setting to a known distance to calibrate WDNI. This calibration was obtained by testing the behavior of the radio with different transmission power settings, and regulating the multi-hop behavior with the help of RSSI filtering. The complete calibration process is omitted here due to space restrictions.

The main results are depicted in Figure 4: Here, the average, normalized error of the distance estimation with WDNI in the three different testbed scenarios is plotted, as well as the dispersion of obtained error values with the help of interquartile diagrams. Once again, WDNI works best for a uniformly distributed network. The average, normalized miscalculation of the nodes of 0.27R in this setting equals to 0.945 m, with the best 25% of the distance calculations having an error below 0.11R or 0.385 m, a value that provides a good accuracy for indoor usage. In 75% of all cases, the error remains at a value of 0.38R or a maximal offset of 1.3 m within acceptable bounds. Both, the horseshoe and the asymmetrical distribution remain below a threshold of 0.2R which is equivalent to 0.7 m in interquartile 1, and feature an average error of roughly 1.1 m at the most, an observation that shows the validity of applying WDNI to near-uniform network distributions despite its initial design for uniform distributions.

V. Evaluation of WDNI

An evaluation of range-free distance estimation algorithms can only be valid when implemented and tested on real hardware. Therefore, we proceed in two steps to critically analyze the quality of WDNI: We first examine the impact of physical effects on the algorithm by comparing simulation and testbed results. In a second step, we compare WDNI to another range-free distance estimation method based on RSSI measurements, both performed in the same network settings, to show that WDNI outperforms common estimation alternatives. A final overview of the error values is displayed in table I to summarize the findings.

A. Simulative vs. Testbed Results

The good results obtained by testing WDNI on `ns-2` are mostly confirmed in the test run conducted with the ScatterWeb sensor nodes. Looking at the distance estimation for a uniform node distribution, the average error is slightly lower in a simulation environment than in an implementation on real hardware. Keep in mind that the SRR value for the experimental setup corresponds to a simulative value between 0.5 and 1 which has to be considered when comparing the overall averages of testbed and simulation results. Values for interquartile 1 are surprisingly better in the testbed, meaning that many sensor nodes are able to estimate their relative position very well. On the down-side however, a great number of miscalculation add to the higher values in the other interquartiles. We account this increase of the error in real

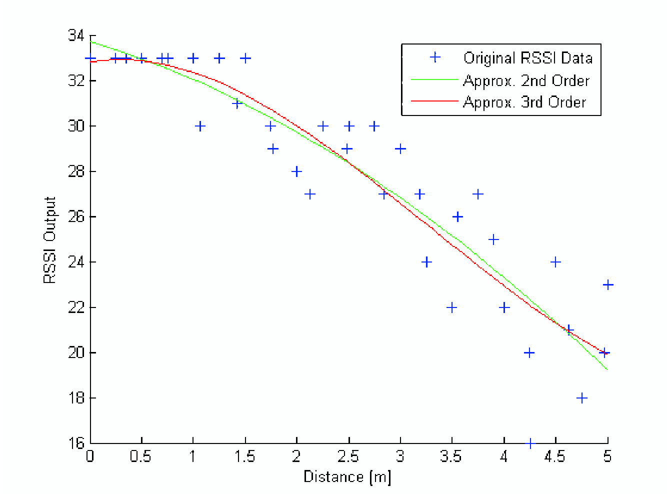

Fig. 5. Approximation of the distance between nodes based on RSSI for indoor scenario at a transmission power of 0x01.

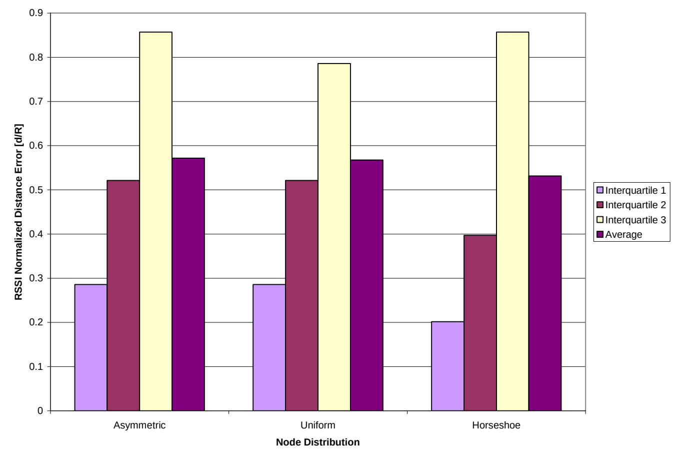

Fig. 6. Averaged normalized errors per interquartile in a asymmetrical, horseshoe and uniform distribution derived from RSSI distance estimation.

scenarios to external influences such as fading, interference or asymmetric links. The discrepancy of results observable in horseshoe and asymmetric network layouts for the two complementary evaluation methods is rather unexpected. Here, the testbed environment performs somewhat better in average and interquartile 1, an insight that we are not able to link to a concrete physical or algorithmic influence.

B. WDNI vs. RSSI-based Range-Free Distance Estimation

The evaluation of the quality of the proposed range-free distance estimation algorithm is complemented by a quantitative comparison to a similar estimation based on RSSI data. To guarantee a fair validation, the same testbed settings including the physical setup of the sensor nodes and the best transceiver settings have been used. Furthermore, the function that maps the strength of the received signal to a specific distance as depicted in Figure 5 has been constructed by interweaving several RSSI measurements with different node positions into a suitable approximation. Note that the validity of the polynom obtained with MatLab

$$f_x = -0.2996x^2 - 1.407x + 33.723 \tag{5}$$

is constrained to the measurement area.

Within the RSSI comparison testbed, each node of the network computes its distance to the other nodes of the network by substituting the value of the received signal strength for x in equation 5 and solving for f_x to obtain the corresponding distance. Figure 6 shows the average normalized error for the different testbed setups. Although the normalization of the results is artificially applied after the collection of the data, it has been chosen for comparability reasons. Results can easily re-mapped to discrete error values by multiplying them by 3.5m.

The data on RSSI distance estimation as shown in Figure 6 reveals the weaknesses of relying solely on RSSI readings. In average, RSSI distance estimation errors are almost twice as high for all three tested scenarios than the ones provided by WDNI, leading to an average mis-placement of 1.96 m in a uniformly distributed network. The approximation simply lacks the required flexibility to cope with the problem of fluctuation of the received signal strength and their low resolution, a problem that WDNI is able to at least partially solve with its exploitation of weighted, local node densities.

VI. Related Work

A plethora of algorithms have been proposed to estimate distances between sensor nodes. The publication having the most similarity with our work is described in [1]. Here, Buschmann et al. present an algorithm to infer distances between nodes inspired by the observation that distant nodes have fewer neighbors in common than close ones. They first use a mathematical model to derive the overlap of two unit disc graphs representing the transmission range, and relate this intersection area to a number of shared neighbors. Depending on this number and different radio models, a look up table to determine the distance between two nodes can be constructed.

Although we also rely on the shared neighbor assumption in our algorithm, we enhance this idea by taking into consideration the local node densities, a factor that improves the approximation. Furthermore, we test the WDNI algorithm not only by means of simulations but on real hardware with ScatterWeb nodes and compare its performance to an RSSI-based range-free distance estimation method, a thorough evaluation not that has not been provided beforehand.

The exploitation of the received signal strength for distance estimation from several points of view. Extensive measurements in both obstacle-free and indoor environments of signal strength properties for a 2.4 GHz CC2420 radio have e.g. been carried out by Lymberopoulos et. al in [4] to provide a detailed analysis of parameters influencing the RSSI value. Conclusions on how to overcome intrinsic problems are left undiscussed and open for future research. Besides proving the fact that RSSI values are closely correlated to environmental

	WDNI real			*RSSI*			*WDNI Sim.*	
	Asym.	Uniform	Horseshoe	Asym.	Uniform	Horseshoe	Uniform	NearUni.
Min. norm. error	0.0085	0.0085	0.0085	0.0164	0.0164	0.0048	0	0
Max. norm. error	0.68	0.66	0.7075	1.6162	1.9285	2.07	0.39	0.42

TABLE I
MINIMAL AND MAXIMAL NORMALIZED ERROR VALUES OBTAINED BY NODES FOR DIFFERENT EXPERIMENTS AND SETTINGS.

parameters, Zhao et. al [11] explore the feasibility of concluding the distance of a transmitting Berkeley Mica Mote operating at 433 MHz from the reception rate of packets. Their main idea is to set up a merit figure based on RSSI to describe and quantify the corresponding reception rate. Furthermore, research in the area of range-free positioning and localization methods include studies on optimizing antennas to provide less fluctuating signals [10] or the development of more sophisticated algorithms relying on ordering and ranking sequences of measurements to predefined reference points to identify unique regions within the localization space. Although the distance estimation method based on RSSI values that we utilized for our comparison is rather simplistic, we based the chosen algorithm on intensive calibrations. With regard to the instability of RSSI values identified, we tried to provide the fairest comparison possible to WDNI.

VII. FUTURE WORK

First of all, it is necessary to evaluate the algorithm in larger testbeds to confirm the good accuracy of WDNI for variable node densities. It will be especially interesting to find out whether a lower bound for the number of neighboring nodes and a given accuracy can be derived.

Furthermore, focusing on different node distribution patterns may lead to better distance estimations in these kinds of networks. Here, future work includes investigating whether approximations as have been presented in this paper, can be retrieved for different non-uniform distributions, and whether their quality yields similar error ranges.

Finally, the main goal will be to use WDNI in the localization context. Therefore, we plan to integrate our algorithm into several approaches for location estimation, such as DV-Hop or APIT [3]. We expect to be able to improve the basic algorithms with transitive evaluation of neighborhood information within the local scope of the nodes.

VIII. CONCLUSIONS

In this paper we presented WDNI, an algorithm to estimate distances between two adjacent nodes based solely on local neighborhood information. As a foundation, the area of intersection of two overlapping transmission ranges has been related to the number of nodes in a uniformly distributed network. In case this general approximation is weighted with local densities of the nodes involved, it can be used to determine their distance.

To investigate the accuracy of the WDNI algorithm, we rely both on simulations and experiments with real sensor nodes and provide an exhaustive evaluation against another range-free distance estimation algorithm. A tendency towards better results for high densities can be observed in the simulations. Here, the best average, normalized error obtained has been 0.16R for a uniform distribution of sensor nodes, with the majority of values for different node densities being below 0.35R. WDNI also works in non-uniform environments, but features slightly higher error values. Different testbed layouts have been used to put WDNI into practice. A lesson learned is that the calibration of the radio transceiver has a high impact on the accuracy of the estimations, thus it is important to carefully judge experimental results. Our measurements reflect the findings of the simulations. With the help of a direct comparison to an RSSI-based range-free distance estimation approach we were able to judge the accuracy of our approach. Here, we were able to show that WDNI not only provides better average accuracy for estimations, but also that the overall value dispersion of these estimations is better.

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