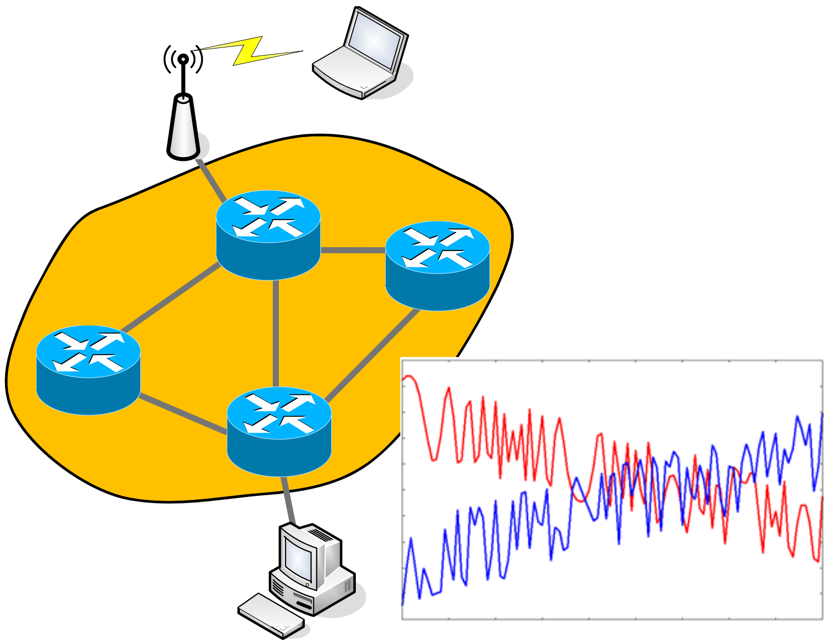
Chapter 2

Simulation Examples



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Simulation using Tables

Simulation using a Table

- Introducing simulation by manually simulating on a table
  - Can be done via pen-and-paper or by using a spreadsheet

Repetition i	Inputs x_{i1}	x_{i2}	…	x_{ij}	…	x_{ip}	Response y_i
1							
2							
:							
n							

- Three steps
  1. Determine the characteristics of each input to the simulation.
  2. Construct a simulation table consisting of
     - p inputs x_{ij}, $j=1,2,\ldots,p$
     - one response y_i, $i=1,2,\ldots,n$
  3. For each repetition i, generate a value for each of the p inputs x_{ij} and calculate the response y_i.

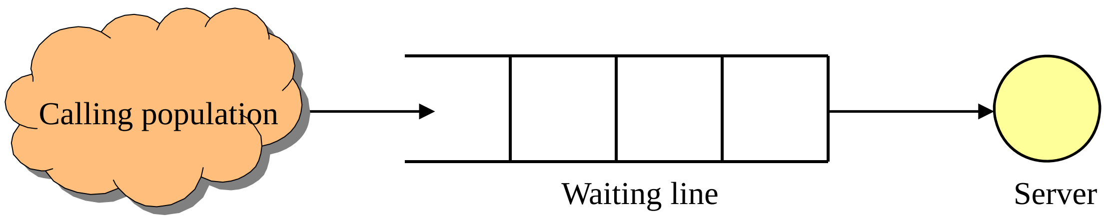
➡ Next some simulation examples using tables.

Simulation of Queueing Systems

Simulation of Queueing Systems: Why?

Simulation of Queueing Systems

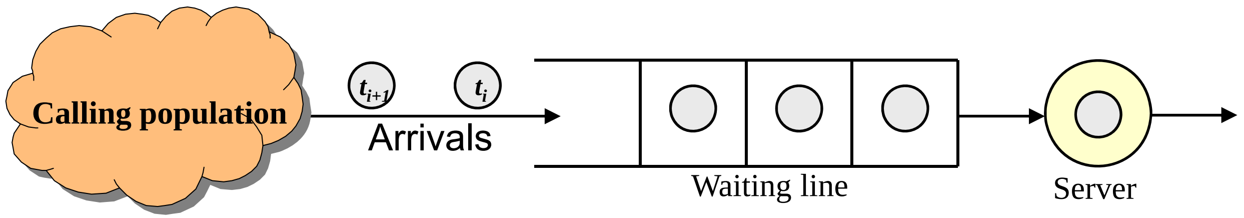
- A queueing system is described by
  - Calling population
  - Arrival rate
  - Service mechanism
  - System capacity
  - Queueing discipline



Simulation of Queueing Systems

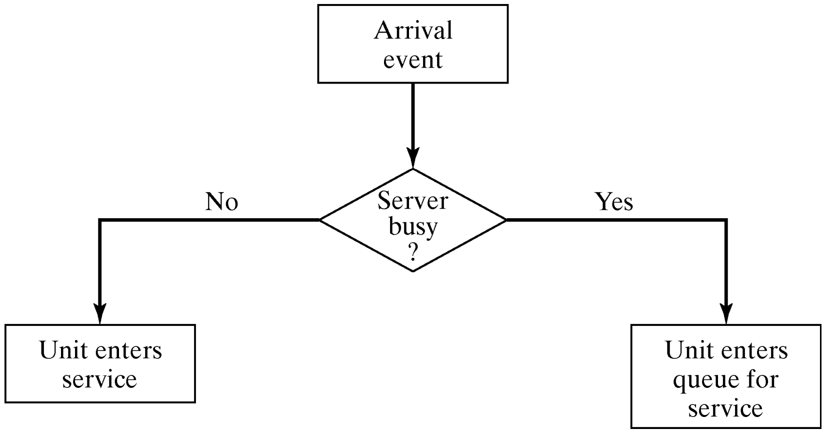
- Single server queue
  - Calling population is infinite
    ➡ Arrival rate does not change
  - Units are served according FIFO
  - Arrivals are defined by the distribution of the time between arrivals ➡ inter-arrival time
  - Service times are according to a distribution
  - Arrival rate must be less than service rate ➡ stable system
  - Otherwise waiting line will grow unbounded ➡ unstable system

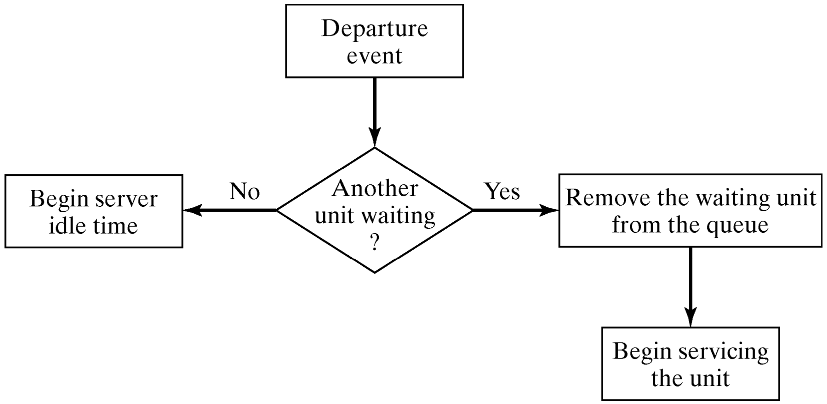
- Queueing system state
  - System
    - Server
    - Units (in queue or being served)
    - Clock
  - State of the system
    - Number of units in the system
    - Status of server (idle, busy)
  - Events
    - Arrival of a unit
    - Departure of a unit



Simulation of Queueing Systems

- Arrival Event
  - If server idle unit gets service, otherwise unit enters queue.
- Departure Event
  - If queue is not empty begin servicing next unit, otherwise server will be idle.





- How do events occur?
  - Events occur randomly
  - Interarrival times $\in \{1,...,6\}$
  - Service times $\in \{1,...,4\}$

Simulation of Queueing Systems

The interarrival and service times are taken from distributions!

Customer	Interarrival Time	Arrival Time on Clock	Service Time
1	-	0	2
2	2	2	1
3	4	6	3
4	1	7	2
5	2	9	1
6	6	15	4

The simulation run is build by meshing clock, arrival, and service times!

Customer Number	Arrival Time [Clock]	Time Service Begins [Clock]	Service Time [Duration]	Time Service Ends [Clock]
1	0	0	2	2
2	2	2	1	3
3	6	6	3	9
4	7	9	2	11
5	9	11	1	12
6	15	15	4	19

Simulation of Queueing Systems

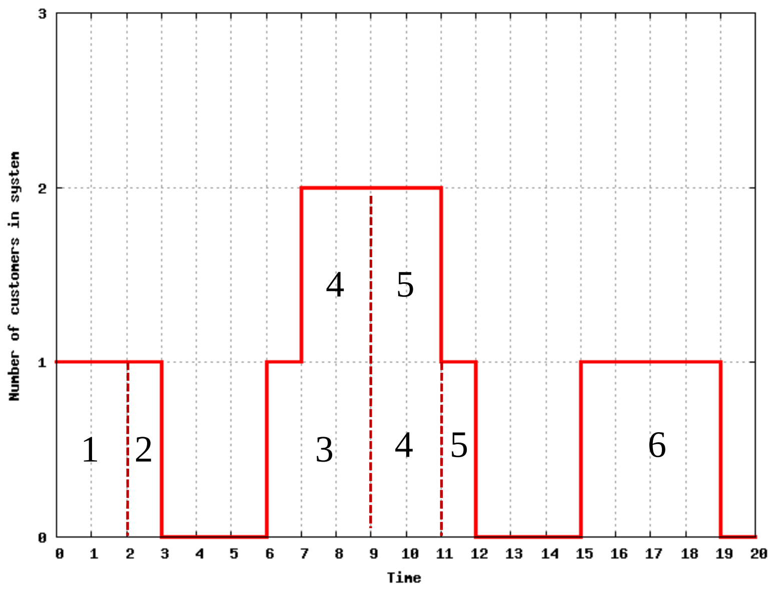
Interesting observations

- Customer 1 is in the system at time 0
- Sometimes, there are no customers
- Sometimes, there are two customers
- Several events may occur at the same time

Chronological ordering of events

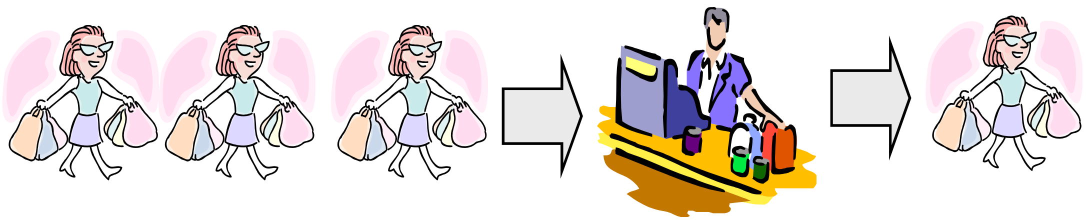
Clock Time	Customer Number	Event Type	Number of customers
0	1	Arrival	1
2	1	Departure	0
2	2	Arrival	1
3	2	Departure	0
6	3	Arrival	1
7	4	Arrival	2
9	3	Departure	1
9	5	Arrival	2
11	4	Departure	1
12	5	Departure	0
15	6	Arrival	1
19	6	Departure	0

Number of customers in the system



Examples

Example 1: A Grocery



Example 1: A Grocery

- Analysis of a small grocery store
  - One checkout counter
  - Customers arrive at random times from {1,2,…,8} minutes
  - Service times vary from {1,2,…,6} minutes
  - Consider the system for 100 customers

Interarrival Time [minute]	Probability	Cumulative Probability
1	0.125	0.125
2	0.125	0.250
3	0.125	0.375
4	0.125	0.500
5	0.125	0.625
6	0.125	0.750
7	0.125	0.875
8	0.125	1.000

Service Time [minute]	Probability	Cumulative Probability
1	0.10	0.10
2	0.20	0.30
3	0.30	0.60
4	0.25	0.85
5	0.10	0.95
6	0.05	1.00

- Problems/Simplifications
  - Sample size is too small to be able to draw reliable conclusions
  - Initial condition is not considered

Example 1: A Grocery

Simulation run for 100 customers

	Simulation System					Performance Measure		
Customer	Interarrival Time [Minutes]	Arrival Time [Clock]	Service Time [Minutes]	Time Service Begins [Clock]	Time Service Ends [Clock]	Waiting Time in Queue [Minutes]	Time Customer in System [Minutes]	Idle Time of Server [Minutes]
1	-	0	4	0	4	0	4	0
2	1	1	2	4	6	3	5	0
3	1	2	5	6	11	4	9	0
4	6	8	4	11	15	3	7	0
5	3	11	1	15	16	4	5	0
6	7	18	5	18	23	0	5	2
...								
100	5	415	2	416	418	1	3	0
Total	415		317			174	491	101

Example 1: A Grocery, Some statistics

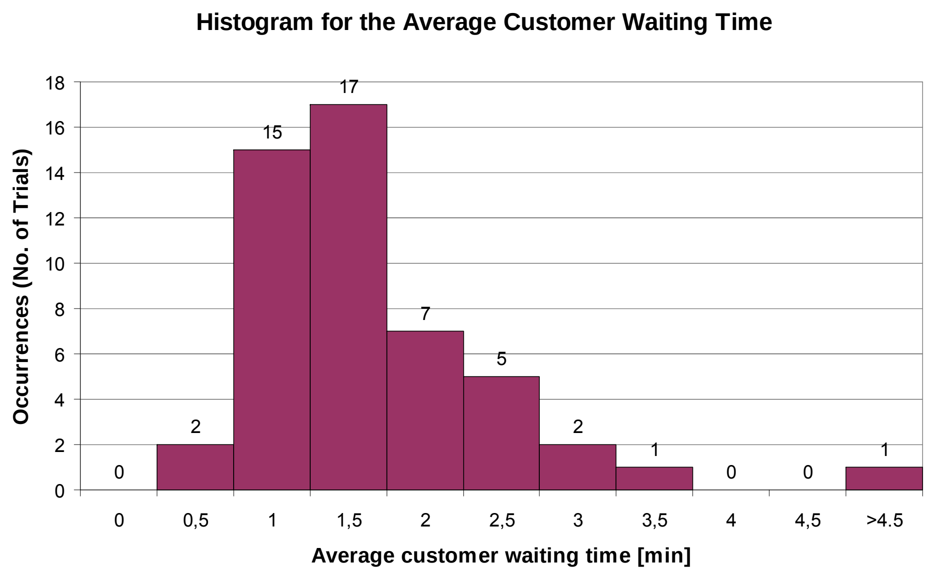
Average waiting time	$\overline{w} = \frac{\sum \text{Waiting time in queue}}{\text{Number of customers}} = \frac{174}{100} = 1.74\,\text{min}$
Probability that a customer has to wait	$p(\text{wait}) = \frac{\text{Number of customer who wait}}{\text{Number of customers}} = \frac{46}{100} = 0.46$
Proportion of server idle time	$p(\text{idle server}) = \frac{\sum \text{Idle time of server}}{\text{Simulation run time}} = \frac{101}{418} = 0.24$
Average service time	$\overline{s} = \frac{\sum \text{Service time}}{\text{Number of customers}} = \frac{317}{100} = 3.17\,\text{min}$ $E(s) = \sum_{s=0}^{\infty} s \cdot p(s) = 0.1 \cdot 10 + 0.2 \cdot 20 + \cdots + 0.05 \cdot 6 = 3.2\,\text{min}$
Average time between arrivals	$\overline{\lambda} = \frac{\sum \text{Times between arrivals}}{\text{Number of arrivals - 1}} = \frac{415}{99} = 4.19\,\text{min}$ $E(\lambda) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5\,\text{min}$
Average waiting time of those who wait	$\overline{w}_{waited} = \frac{\sum \text{Waiting time in queue}}{\text{Number of customers that wait}} = \frac{174}{54} = 3.22\,\text{min}$
Average time a customer spends in system	$\bar{t} = \frac{\sum \text{Time customers spend in system}}{\text{Number of customers}} = \frac{491}{100} = 4.91\,\text{min}$ $\bar{t} = \overline{w} + \overline{s} = 1.74 + 3.17 = 4.91\,\text{min}$

Example 1: A Grocery, Some statistics

- Interesting results for a manager, but
  - longer simulation run would increase the accuracy

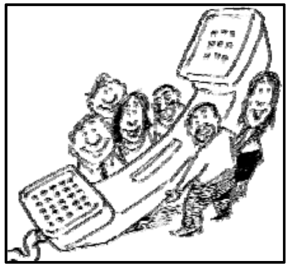
- Some interpretations
  - Average waiting time is not high
  - Server has not undue amount of idle time, it is well loaded ;-)
  - Nearly half of the customers have to wait (46%)

Average of 50 Trials



Examples

Example 2: Call center



Example 2: Call Center Problem

- Consider a Call Center where technical personnel take calls and provide service
- Two technical support people (2 server) exists
  - Able more experienced, provides service faster
  - Baker newbie, provides service slower
- Rule
  - Able gets call if both people are idle
  - Try other rules
    - Baker gets call if both are idle
    - Call is assigned randomly to Able and Baker
- Goal of study: Find out how well the current rule works

Example 2: Call Center Problem

- Interarrival distribution of calls for technical support

Time between Arrivals [Minute]	Probability	Cumulative Probability	Random-Digit Assignment
1	0.25	0.25	01 – 25
2	0.40	0.65	26 – 65
3	0.20	0.85	66 – 85
4	0.15	1.00	86 – 00

Service time distribution of Baker

Service Time [Minute]	Probability	Cumulative Probability	Random-Digit Assignment
3	0.35	0.35	01 – 35
4	0.25	0.60	36 – 60
5	0.20	0.80	61 – 80
6	0.20	1.00	81 – 00

Service time distribution of Able

Service Time [Minute]	Probability	Cumulative Probability	Random-Digit Assignment
2	0.30	0.30	01 – 30
3	0.28	0.58	31 – 58
4	0.25	0.83	59 – 83
5	0.17	1.00	84 – 00

Example 2: Call Center Problem

Simulation proceeds as follows

- Step 1:
  - For Caller k, generate an interarrival time A_k. Add it to the previous arrival time T_{k-1} to get arrival time of Caller k as $T_k = T_{k-1} + A_k$
- Step 2:
  - If Able is idle, Caller k begins service with Able at the current time T_{now}
  - Able's service completion time $T_{fin,A}$ is given by $T_{fin,A} = T_{now} + T_{svc,A}$ where $T_{svc,A}$ is the service time generated from Able's service time distribution. Caller k's waiting time is $T_{wait} = 0$.
  - Caller k's time in system, T_{sys}, is given by $T_{sys} = T_{fin,A} – T_k$
  - If Able is busy and Baker is idle, Caller begins with Baker. The remainder is in analogous.
- Step 3:
  - If Able and Baker are both busy, then calculate the time at which the first one becomes available, as follows: $T_{beg} = \min(T_{fin,A}, T_{fin,B})$
  - Caller k begins service at T_{beg}. When service for Caller k begins, set $T_{now} = T_{beg}$.
  - Compute $T_{fin,A}$ or $T_{fin,B}$ as in Step 2.
  - Caller k's time in system is $T_{sys} = T_{fin,A} – T_k$ or $T_{sys} = T_{fin,B} - T_k$

Example 2: Call Center Problem

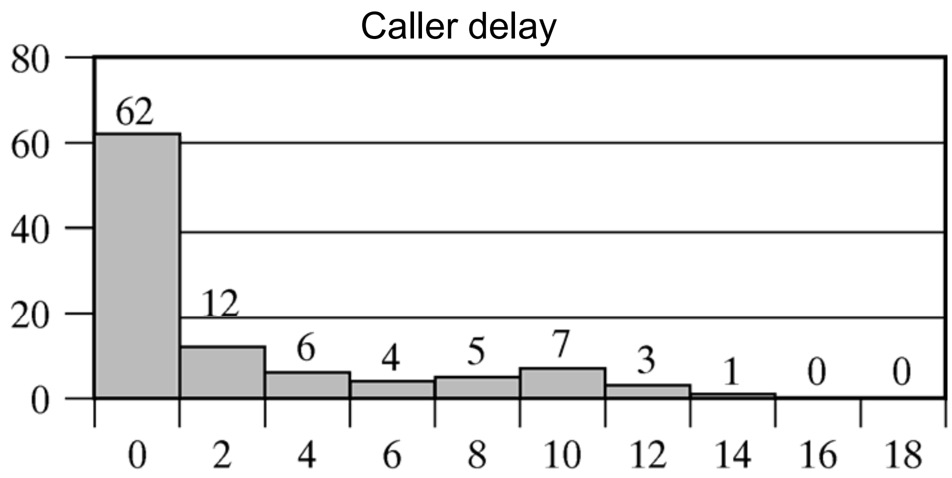
- Simulation run for 100 calls

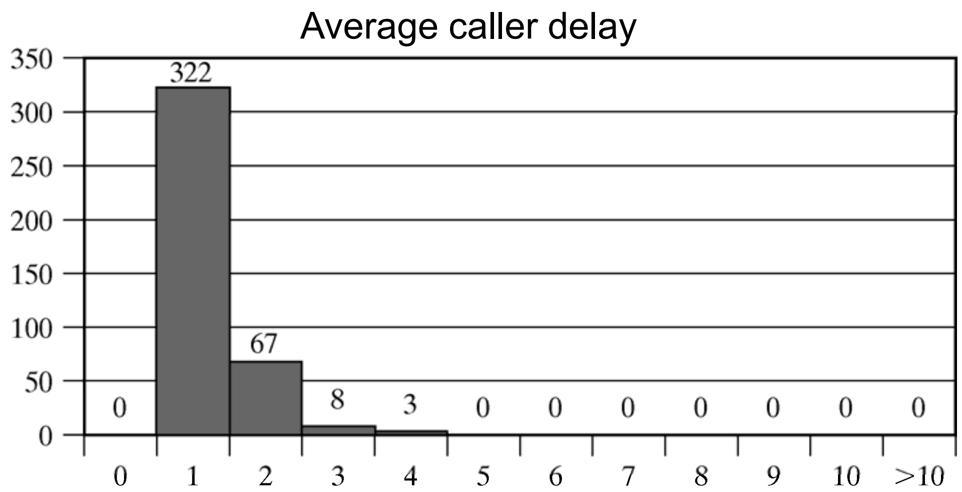
Caller Nr.	Interarrival Time	Arrival Time	When Able Avail.	When Baker Avail.	Server Chosen	Service Time	Time Service Begins	Able's Service Compl. Time	Baker's Service Compl. Time	Caller Delay	Time in System
1	-	0	0	0	Able	2	0	2		0	2
2	2	2	2	0	Able	2	2	4		0	2
3	4	6	4	0	Able	2	6	8		0	2
4	2	8	8	0	Able	4	8	12		0	4
5	1	9	12	0	Baker	3	9		12	0	3
...	...	...	...	...							
100	1	219	221	219	Baker	4	219			0	4
Total										211	564

Example 2: Call Center Problem, Some statistics

- One simulation trial of 100 caller
  - 62% of callers had no delay
  - 12% of callers had a delay of 1-2 minutes

- 400 simulation trials of 100 caller
  - 80.5% of callers had delay up to 1 minute
  - 19.5% of callers had delay more than 1 minute



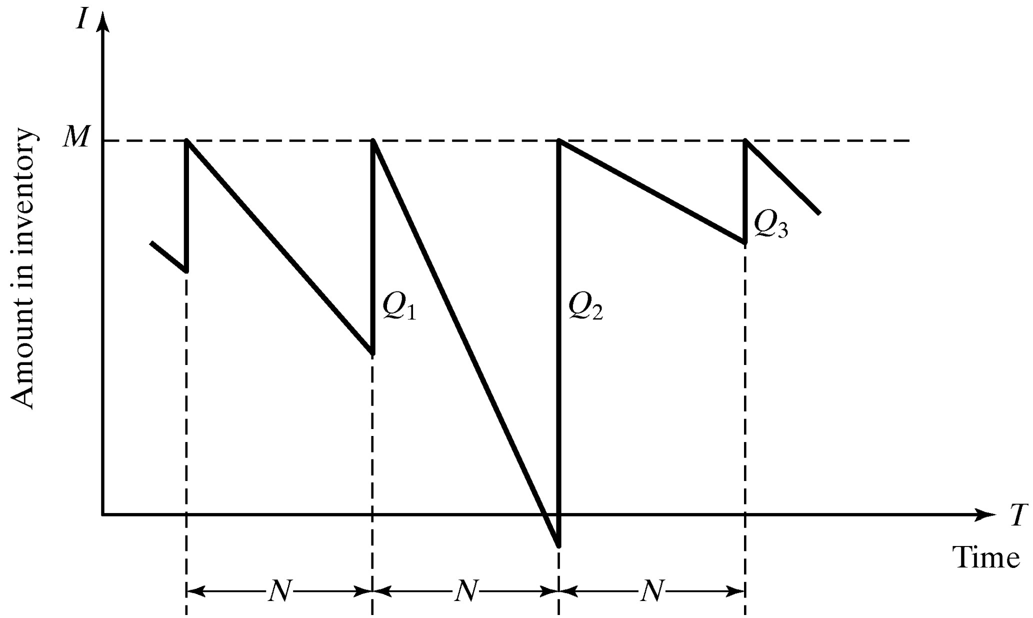


Examples

Example 3: Inventory System

Example 3: Inventory System

- Important class of simulation problems: inventory systems
- Performance measure: Total cost (or total profit)
- Parameters
  - N Review period length
  - M Standard inventory level
  - Q_i Quantity of order i to fill up to M



Example 3: Inventory System

- Events in an (M, N) inventory system are
  - Demand for items
  - Review of the inventory
  - Receipt of an order at the end of each review

- To avoid shortages, a buffer stock is needed
- Cost of stock
  - Storage space
  - Guards

Appendix

Random digits

Appendix: Random Digits

- Producing random numbers from random digits
  - Select randomly a number, e.g.
    - One digit: 0.9
    - Two digits: 0.19
    - Three digits: 0.219
  - Proceed in a systematic direction, e.g.
    - first down then right
    - first up then left

94737	08225	35614	24826	88319	05595	58701	57365	74759
87259	85982	13296	89326	74863	99986	68558	06391	50248
63856	14016	18527	11634	96908	52146	53496	51730	03500
66612	54714	46783	61934	30258	61674	07471	67566	31635
30712	58582	05704	23172	86689	94834	99057	55832	21012
69607	24145	43886	86477	05317	30445	33456	34029	09603
37792	27282	94107	41967	21425	04743	42822	28111	09757
01488	56680	73847	64930	11108	44834	45390	86043	23973
66248	97697	38244	50918	55441	51217	54786	04940	50807
51453	03462	61157	65366	61130	26204	15016	85665	97714
92168	82530	19271	86999	96499	12765	20926	25282	39119
36463	07331	54590	00546	03337	41583	46439	40173	46455
47097	78780	04210	87084	44484	75377	57753	41415	09890
80400	45972	44111	99708	45935	03694	81421	60170	58457
94554	13863	88239	91624	00022	40471	78462	96265	55360
31567	53597	08490	73544	72573	30961	12282	97033	13676
07821	24759	47266	21747	72496	77755	50391	59554	31177
09056	10709	69314	11449	40531	02917	95878	74587	60906
19922	37025	80731	26179	16039	01518	82697	73227	13160
29923	02570	80164	36108	73689	26342	35712	49137	13482
29602	29464	99219	20308	82109	03898	82072	85199	13103
94135	94661	87724	88187	62191	70607	63099	40494	49069
87926	34092	34334	55064	43152	01610	03126	47312	59578
85039	19212	59160	83537	54414	19856	90527	21756	64783
66070	38480	74636	45095	86576	79337	39578	40851	53503
78166	82521	79261	12570	10930	47564	77869	16480	43972
94672	07912	26153	10531	12715	63142	88937	94466	31388
56406	70023	27734	22254	27685	67518	63966	33203	70803
67726	57805	94264	77009	08682	18784	47554	59869	66320
07516	45979	76735	46509	17696	67177	92600	55572	17245
43070	22671	00152	81326	89428	16368	57659	79424	57604
36917	60370	80812	87225	02850	47118	23790	55043	75117
03919	82922	02312	31106	44335	05573	17470	25900	91080
46724	22558	64303	78804	05762	70650	56117	06707	90035
16108	61281	86823	20286	14025	24909	38391	12183	89393
74541	75808	89669	87680	72758	60851	55292	95663	88326
82919	31285	01850	72550	42986	57518	01159	01786	98145
31388	26809	77258	99360	92362	21979	41319	75739	98082
17190	75522	15687	07161	99745	48767	03121	20046	28013
00466	88068	68631	98745	97810	35886	14497	90230	69264

Summary

- This chapter introduced simulation concepts by means of examples
- Example simulations were performed on a table manually
  - Use a spreadsheet for large experiments (Excel, OpenOffice)
  - Input data is important
  - Random variables can be used
  - Output analysis important and difficult
  - The used tables were of ad hoc, a more methodic approach is needed