HOL based Universal Reasoning

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At Unilog'2010 I have proposed classical higher-order logic HOL (Church's type theory [1,9]) as a uniform framework for combining logics [2]. The main claim has been that many non-classical logics are just natural (embedded) fragments of classical HOL. The approach also supports combinations (e.g. fusions) of embedded logics; in particular, bridge rules can be postulated simply as axioms. In the Unilog'2010 presentation the focus has been on combinations of quantified multimodal logics [7] (quantified wrt first-order and propositional variables) and I have claimed that the approach supports automation of both reasoning *within* and *about* embedded logics and their combinations with off-the-shelf higher-order automated theorem provers (cf. the experiments in [3]).

Significant further progress has been made since the Unilog'2010 event. For example, semantic embeddings for propositional and quantified conditional logics have been added [4,5]. This is particularly interesting since selection function semantics for conditional logics, which is what we have studied, can be seen as a higher-order extension of Kripke semantics for modal logics and cannot be naturally embedded into first-order logic.

Moreover, important and timely application directions of the approach have been identified. Amongst these is the mechanization and automation of expressive ontologies such as SUMO (or Cyc), whose representation languages support combinations of first-order and even higher-order constructs with various modal operators [8]. Related ongoing work also studies HOL embeddings of the semantic web language OWL2-full and the DOLCE ontology.

Practical effectiveness of the HOL based universal reasoning approach can be evaluated by comparing it with implemented competitor approaches. To date, however, there are no such (implemented) systems available. Only for first-order monomodal logics, which are not in our primary interest since effective specialist reasoners for these logics can still be comparably easily achieved, some automated provers have meanwhile been implemented. A recent evaluation [6] confirms that the HOL based universal reasoning approach, which performed second best in this study, is competitive.

At Unilog'2013 I will present and discuss the project progress as sketched above. I will also demonstrate our HOL based universal reasoner, which calls various off-the-shelf HOL theorem provers and model finders remotely. Moreover, I will outline future directions of this research.

References

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