Einstein Workshop on
Polytopes and Algebraic Geometry

Titles and abstracts

December 2-4, 2019

Freie Universität Berlin

Einstein Stiftung Berlin
Einstein Foundation Berlin

Program

	Monday Dec. 2 (ZIB)	**Tuesday Dec. 3** (ZIB)	**Wednesday Dec. 4** (Hotel Steglitz Int.)
08.30 - 09.00	*Registration*		
09.00 - 10.00	**Benjamin Braun**	**Johannes Hofscheier**	**Winfried Bruns**
10.00 - 10.30	*Coffee break*		
10.30 - 10.55	Andrés R. Vindas Meléndez	Andrea Petracci	Alessio D'Alì
11.00 - 11.25	Davide Bolognini	María-Cruz Fernández-Fernández	Eliana Duarte
11.30 - 11.55	Carlos Alejandro Alfaro Montufar	Oguzhan Yürük	Daniel Tamayo Jiménez
12.00 - 14.00	*Lunch*		
14.00 - 15.00	**Alex Fink**	**Thomas Kahle**	**Alexander Borisov**
15.10 - 15.35	Filip Cano Córdoba	Karin Schaller	Marta Panizzut
15.35 - 16.10	*Coffee break*		
16.10 - 17.10	**Matthias Beck**	**Matthias Schymura**	**Fatemeh Mohammadi**

Minimum rank and critical ideals of graphs

2 Dec
11:30am

Carlos Alejandro Alfaro Montufar
Banco de Mexico

Given a graph G and a set of indeterminates $X_G = x_u : u \in V(G)$, let $R[X_G]$ denote the polynomial ring over a commutative ring R with unity in the variables X_G. We consider the determinantal ideals over $R[X_G]$ of the matrix $A_X(G) = diag(X_G) - A(G)$, where $A(G)$ is the adjacency matrix of G. These ideals are called *critical ideals*, since they are a generalization of the critical group and the Smith group of a graph. And, the varieties associated to these ideals can be regarded as a generalization of the Laplacian and adjacency spectrum. In this talk, we are going to see how these ideals are related with the minimum rank of a graph.

Weighted lattice point sums in lattice polytopes

2 Dec
4:10pm

Matthias Beck
San Francisco State University and Freie Universität Berlin

Let V be a real vector space of dimension n and let $M \subset V$ be a lattice. Let $P \subset V$ be an n-dimensional polytope with vertices in M, and let φ be a homogeneous polynomial function on V of degree d. For a positive integer q and a face F of P, let $D_{\varphi,F}(q)$ be the sum of φ over the lattice points in the dilate qF. We define a generating function $G_\varphi(q,y)$ packaging together the various $D_{\varphi,F}(q)$, and show that it satisfies a functional equation that simultaneously generalizes Ehrhart—Macdonald reciprocity and the Dehn—Sommerville relations. When P is a simple (i.e., each vertex meets n edges), G_φ can be computed using an analogue of Brion—Vergne's Euler—Maclaurin summation formula using Todd operators.
This is joint work with Paul Gunnels (UMass Amherst) and Evgeny Materov (Siberian Fire and Rescue Academy of EMERCOM of Russia)

Chordality and Simon's conjecture

2 Dec
11:00am

Davide Bolognini
Università di Bologna

We recall a notion of chordality for hypergraphs introduced by Bigdeli et al. Despite the fact that this notion does not allow to extend Fröberg's theorem relating chordality of graphs and linear resolution of monomial ideals generated in degree two, they stated a conjecture ensuring the chordality of some hypergraphs associated to shellable simplicial complexes. This conjecture would imply the long-standing open Simon's conjecture on extendable shellability for the skeleta of a simplex.
In this talk we disprove the stronger conjecture, providing an infinite family of shellable simplicial complexes whose associated hypergraphs are not chordal.
This is a joint work with Bruno Benedetti.

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3 Dec
9:00am

Lattice simplices with few lattice points inside: the birational geometry connection

Alexander Borisov
Binghamton University

While being at their core purely combinatorial objects, lattice simplices with few or no lattice points inside have been of keen interest to birational geometers, through the theory of toric varieties. In my talk I will explain how toric varieties are constructed from these lattice simplices, and what they have to do with both Fields medals in birational algebraic geometry: Mori, 1990 and Birkar, 2018.

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2 Dec
9:00am

Algebraic aspects of lattice simplices

Benjamin Braun
University of Kentucky

Given a lattice polytope P, there are open problems of interest related to the integer decomposition property, Ehrhart h^*-unimodality, Ehrhart positivity, and rationality of the Poincare series for the associated semigroup algebra. In this talk, we will survey some recent results in this area with a focus on lattice simplices. This talk is based on various joint work with Brian Davis, Rob Davis, Morgan Lane, Fu Liu, and Liam Solus.

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4 Dec
9:00am

Wilf's conjecture by multiplicity

Winfried Bruns
Universität Osnabrück

Let S be a numerical semigroup. Its embedding dimension $e(S)$ is the minimal number of generators, the Frobenius number $F(S)$ is the largest integer $\notin S$, and $n(S)$ counts the elements in S that are $< F(S)$. Wilf's conjecture states that $F(S) < e(S)n(S)$.
It has been proved in many cases, but remains a major open problem in the combinatorial theory of numerical semigroups. We will show that for fixed multiplicity $m = m(S)$, the smallest nonzero element of S, the conjecture can be decided algorithmically by polyhedral methods using the parametrization of multiplicity m semigroups by the lattice points of the Kunz polyhedron P_m. With them we have verified the conjecture for $m \leq 18$.
This is joint work with Pedro Garcia-Sanchez, Christopher O'Neill and Dane Wilburne.

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2 Dec
3:10pm

Ehrhart positivity of generalized permutohedra

Filip Cano Córdoba
Universitat Politècnica de Catalunya

We present a local formula for the k-th coefficient of the Ehrhart polynomials of a given polytope. This is used to study positivity of said coefficients for regular and generalized permutohedra. We also present the computational difficulties of this approach and ideas to solve them.

Symmetric edge polytopes and their application

4 Dec
10:30am

Alessio D'Alì
University of Warwick

Symmetric edge polytopes are certain lattice polytopes arising from the data of a finite simple graph. In the present talk we introduce some of the pleasant combinatorial properties of these objects and explore some surprising connections to the Kuramoto synchronization model in physics and to the theory of finite metric spaces. If time permits, we will focus on the algebraic-combinatorial tools used to investigate several invariants of interest of these polytopes, e.g. facets and triangulations. This is joint work with E. Delucchi and M. Michałek.

Gröbner bases for staged trees

4 Dec
11:00am

Eliana Duarte
Max-Plack-Institute MIS Leipzig

In this talk we consider the problem of finding generators of the toric ideal associated to a combinatorial object called a staged tree. Our main result states that toric ideals of staged trees that are balanced and stratified are generated by a quadratic Gröbner basis whose initial ideal is squarefree. The proof of this result is based on Sullivant's toric fiber product construction. This talk will be self-contained and several examples and connections to Algebraic Statistics will be highlighted.

On the rank of an A-hypergeometric D-module versus the normalized volume of A

3 Dec
11:00am

María-Cruz Fernández-Fernández
Universidad de Sevilla

This is a joint work with Christine Berkesch. The A-hypergeometric D-module is a system of linear partial differential equations in several complex variables, introduced by Gelfand, Graev, Kapranov and Zelevinsky, that depends on an integer matrix A and a complex parameter vector b. By normalized volume of the matrix A we mean the normalized volume of the lattice polytope defined by the convex hull of the columns of A and the origin. The rank of the D-module is the dimension of its space of holomorphic solutions. When b is generic the rank of the A-hypergeometric D-module is the normalized volume of the matrix A by a result of Adolphson. In general this is known to be only a lower bound and the rank for special b can be much higher than the volume of A. We prove that an upper bound for the quotient rank/volume (valid when b is rank jumping but generic inside the rank jumping locus) is sharp by constructing a family of examples of matrices.

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2 Dec
2:00pm

Newton polytopes of Schubert polynomials

Alex Fink

Queen Mary University of London

We show that the support of a Schubert polynomial is the set of lattice points of a generalised permutohedron. The techniques extend to some related combinatorial polynomials from flag varieties, such as Demazure's key polynomials. We also characterise when a Schubert polynomial has all coefficients 0 or 1. This is joint work with Karola Mészáros and Avery St. Dizier.

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4 Dec
2:00pm

The cohomology ring of toric bundles

Johannes Hofscheier

University of Nottingham

Khovanskii and Pukhlikov observed that the cohomology ring of smooth projective toric varieties is completely described by the Bernstein-Kushnirenko theorem via the volume polynomial on the space of polytopes. In this talk, I will report on joint work with Khovanskii and Monin where we extend this approach to a description of the cohomology ring of equivariant compactifications of torus principal bundles by formulating respective Bernstein-Kushnirenko-type theorems. We conclude the presentation by relating this description of the cohomology ring to a representation of the ring of conditions for horospherical homogeneous spaces.

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3 Dec
2:00pm

The geometry of gaussoids

Thomas Kahle

Otto-von-Guericke-Universität Magdeburg

Gaussoids are combinatorial structures that encode independence in probability and statistics, just like matroids encode independence in linear algebra. We show that the gaussoid axioms of Lnenicka and Matus are equivalent to compatibility with certain quadratic relations among principal and almost-principal minors of a symmetric matrix. This approach facilitates insights into symmetry and realizability of gaussoids as well as several extensions (like oriented, positive, and valuated gaussoids).
Based on joint work with T. Boege, A. D'Ali, and B. Sturmfels

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Toric degenerations of Grassmannians

4 Dec
4:10pm

Fatemeh Mohammadi
University of Bristol

Toric varieties are popular objects in algebraic geometry. This is mainly because they can be modelled on polytopes and polyhedral fans, and there is a dictionary between their geometric properties and the combinatorial invariants of their polytopes. This dictionary can be extended from toric varieties to arbitrary varieties through toric degenerations.
In this talk, I will introduce the notion of toric degenerations which generalizes the fruitful correspondence between toric varieties and polytopes, to arbitrary varieties. There are prototypic examples of toric degenerations (of Grassmannians) which are related to Young tableaux and Gelfand-Cetlin polytopes. I will describe how to obtain such degenerations using the theory of Gröbner fans and tropical geometry. In particular, I will explain a construction of toric degenerations of Grassmannians whose combinatorics are governed by a family of tableaux including the classical semi-standard Young tableaux as an example.

An octanomial model for cubic surfaces

4 Dec
3.10pm

Marta Panizzut
Technische Universität Berlin

Arthur Cayley and George Salmon proved in the 1840s that every smooth cubic surface in $\mathbb{P}^3$ contains exactly 27 lines. Since the early development of tropical geometry, two natural problems were to understand whether the same statement holds for the tropicalization of cubic surfaces and to classify the combinatorial positions of their tropical lines. The answer to the first turned out to be false, as smooth tropical surfaces might contain families of tropical lines. Moreover, classifying positions of tropical lines reveals some computational challenges due to the massive number of combinatorial types of tropical cubic surfaces.
Alternatively, Ren, Shaw and Sturmfels considered the intrinsic tropicalization of very affine surfaces obtained from cubic surfaces by removing the 27 lines. They identified two generic types of such tropical surfaces characterized by their structure at infinity, which is an arrangement of 27 trees with 10 leaves.
In this talk we will tell this tropical story and introduce an octanomial model for cubic surfaces. This new normal form is well suited for p-adic geometry, as it reveals the intrinsic del Pezzo combinatorics of the 27 trees in the tropicalization. The talk is based on joint work with Emre Sertöz and Bernd Sturmfels.

Deformations of toric Fano 3-folds via Minkowski deformations and Cayley sum

3 Dec
10:30am

Andrea Petracci
Freie Universität Berlin

There is a well-known dictionary for many algebro-geometric properties of toric varieties in terms of combinatorial properties of their associated fans, but the deformation theory of (non-affine) toric varieties is still largely unknown. In this talk, following pioneering works by Klaus Altmann, I will show how Minkowski decompositions (and their associated Cayley sums) of all the facets of a reflexive 3-dimensional polytope P induce deformations of the toric Fano 3-fold associated to the face fan of P. This is joint work with Alessio Corti and Paul Hacking.

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3 Dec
3.10pm

Minimal models of surfaces with $p_g = 1$ associated with canonical Fano 3-polytopes

Karin Schaller

Freie Universität Berlin

Let Δ be a canonical Fano 3-polytope, i.e., a 3-dimensional lattice polytope containing exactly one interior lattice point. Then the affine surface Z_Δ defined by a generic Laurent polynomial f_Δ with the Newton polytope Δ is birational to a smooth projective minimal surface S_Δ with $p_g = 1$. Using the classification of all 674,688 canonical Fano 3-polytopes obtained by Kasprzyk together with the notion of the Fine interior Δ^{FI} of Δ, we show that S_Δ is a $K3$-surface except for exactly 9,089 canonical Fano 3-polytopes Δ. In the latter case, we obtain 9,040 canonical Fano 3-polytopes Δ defining minimal elliptic surfaces S_Δ of Kodaira dimension 1 and 49 canonical Fano 3-polytopes Δ defining minimal surfaces S_Δ of general type with $|\pi_1(S_\Delta)| = K^2 \in \{1,2\}$. This is a joint work with Alexander Kaspzryk and Victor Batyrev.

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3 Dec
4:10pm

Tropical volume by tropical Ehrhart polynomials

Matthias Schymura

École Polytechnique Fédérale de Lausanne

Motivated by a recent demand for metric understanding of tropical polytopes, we introduce a natural concept of volume in tropical geometry. This is achieved by developing the foundations of a tropical analog of Ehrhart theory, that is, lattice point counting in dilates of polytopes. We exhibit the basic properties of our volume concept, compare it to existing measures, and discuss arising complexity questions.
Based on joint work with Georg Loho.

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4 Dec
11:30am

Inversion and cubic vectors for permutrees

Daniel Tamayo Jiménez

Université Paris-Sud

Permutrees are combinatorial objects that generalize and amalgam binary trees, permutations, binary words, and Cambrian trees. In this talk we will give a quick introduction to permutrees and introduce two generalizations of the bracket vectors of binary trees. We will see how these generalizations encode algebraic and geometric properties of the rotation lattice on permutrees. In particular, how they lead to a meet operation between permutrees and the construction of a cubical complex for each permutreehedra.

Equivariant Ehrhart theory of the permutahedron

2 Dec
10:30am

Andrés R. Vindas Meléndez
University of Kentucky

In 2010, Stapledon described a generalization of Ehrhart theory with group actions. In 2018, Ardila, Schindler, and I made progress towards answering one of Stapledon's open problems that asked to determine the equivariant Ehrhart theory of the permutahedron. We proved some general results about the fixed polytopes of the permutahedron, which are the polytopes that are fixed by acting on the permutahedron by a permutation. In particular, we computed their dimension, showed that they are combinatorially equivalent to permutahedra, provided hyperplane and vertex descriptions, and proved that they are zonotopes. Lastly, we obtained a formula for the volume of these fixed polytopes, which is a generalization of Richard Stanley's result of the volume for the standard permutahedron. Building off of the work of the aforementioned, we determine the equivariant Ehrhart theory of the permutahedron, thereby resolving the open problem. This project presents combinatorial formulas for the Ehrhart quasipolynomials and Ehrhart Series of the fixed polytopes of the permutahedron, along with other results regarding interpretations of the equivariant analogue of the Ehrhart series. This is joint work with Federico Ardila (San Francisco State University) and Mariel Supina (UC Berkeley).

Understanding the parameter regions of multistationarity in dual phosporylation cycle via SONC

3 Dec
11:30am

Oguzhan Yürük
Technische Universität Braunschweig

Parameterized ordinary differential equation systems are crucial for modeling in biochemical reaction networks under the assumption of mass-action kinetics. Existence of multiple positive solutions in networks arising from biochemical systems underlies switch-like responses in cellular decision making. However, it is difficult to determine whether and when multistationarity exists in a reaction network in general. Recent developments points out that the multistationarity, along with some other qualitative properties of the solutions, is related to various questions concerning the signs of multivariate polynomials in positive orthant. In this work, we provide further insight to the set of kinetic parameters that enable or preclude multistationarity of dual phosphorylation cycle by utilizing circuit polynomials to find symbolic certificates of nonnegativity.
This is a joint work with Elisenda Feliu, Nidhi Kaihnsa and Timo de Wolff.

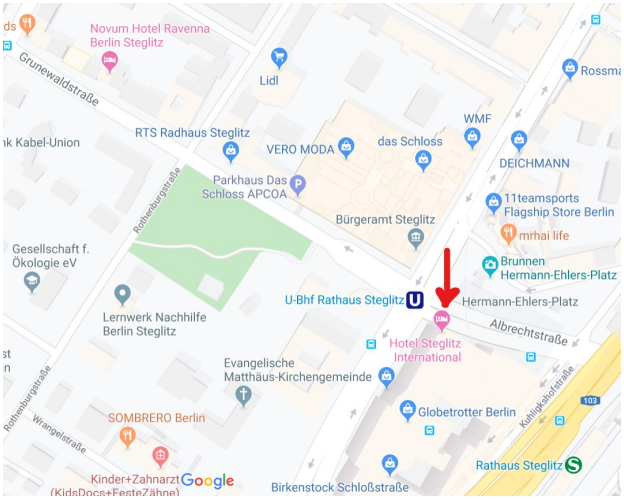
Conference Venues

Zuse Institute Berlin (ZIB)
Takustraße 7, 14195 Berlin

How to get to ZIB with public transportation:

- With U3 get off at Dahlem-Dorf and walk 9 minutes (700 m) or take the X83 bus in direction Marienfelde, Nahmitzer Damm and get off after two stops at Arnimallee.
- With U9 or S-Bahn get off at Rathaus Steglitz and take the bus X83 in direction Königin-Luise-Str./Clayallee and get off after four stops at Arnimallee.
- If you are staying at the Hotel Steglitz International or at the Novum Hotel Ravenna Berlin Steglitz, take the X83 bus in direction Königin-Luise-Str./Clayallee and get off at Arnimallee.

Hotel Steglitz International
Albrechtstrasse 2, 12165 Berlin



How to get to the Hotel Steglitz International with public transportation:

- With U3 get off at Dahlem-Dorf, take the bus X83 in direction Marienfelde, Nahmitzer Damm and get off after six stops at Rathaus Steglitz.
- With U9 or S-Bahn get off at Rathaus Steglitz.
- If you are staying at the Novum Hotel Ravenna Berlin Steglitz, walk 4 minutes (350 m).

Restaurants near the Zuse Institute

Cafeteria (Mensa) ⚠**No** cash and credit card payments! Need to buy a mensa card
Otto-von-Simson-Straße 26, `www.stw.berlin/mensen/mensa-fu-ii.html`

Fast-food

- **Asia Snack Dahlem** | *Vietnamese* [link]
  Königin-Luise-Straße 38
- **Really Good Life** | *Burger* [link]
  Königin-Luise-Straße 44
- **Cantine of Julius Kühn Institute** | *German Cantine*
  Königin-Luise-Straße 19

Bio-cafe with vegetarian options

- **Baci's Coffee** | Coffee (with espresso machine) [link]
  Königin-Luise-Straße 39
- **Cafeteria im Museum** | *Ethnological*
  Lansstraße 8

Sit-down restaurants

- **Alter Krug** | *German* [link]
  Königin-Luise-Straße 52
  Phone: +49 30 832 700 0
- **Luise** | *German* [link]
  Königin-Luise-Straße 40
  Phone: +49 30 841 888 0
- **Ristorante Piaggio** | *Italian* [link]
  Königin-Luise-Straße 44
  Phone: +49 30 832 022 66
- **Restaurant Englers** | *German & French (Fancy)* [link]
  Englerallee 42
  Phone: +49 30 303 642 36

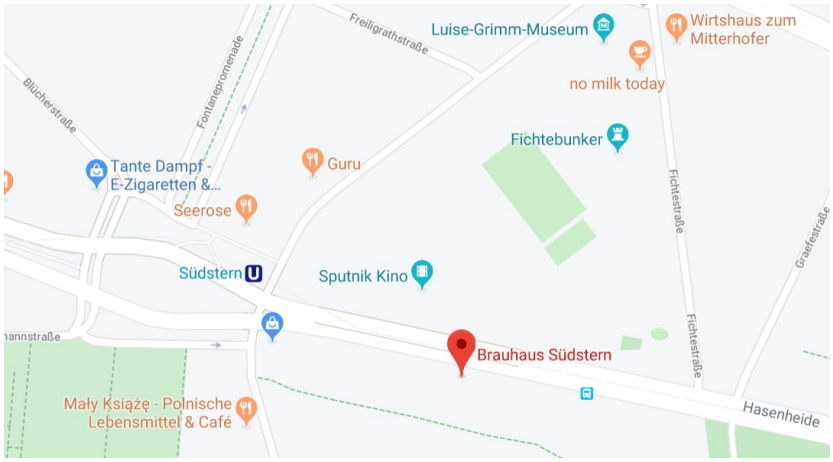
Other suggested restaurants easily reachable from FU

- **Malafemmena** | *Italian (Neapolitan pizza)* [link]
  Hauptstraße 85
  Phone: +49 30 841 831 82
- **Bahadur** | *Indian* [link] (near Fehrbelliner Platz)
  Sigmaringer Straße 36
  Phone: +49 30 224 746 10
- **Shaniu's House of Noodles** | *Chinese* [link]
  Pariser Straße 58
  Phone: +49 30 915 526 05

For Wednesday, the Hotel Steglitz International is located in a commercial area with many lunch options nearby.

Social dinner - Monday, 2 December 2019 at 19:00h

Brauhaus Südstern
Hasenheide 69, 10967 Berlin
brauhaus-suedstern.de



The social dinner, excluding beverages, will be paid by the workshop.
The brewery is located near the U7 metro station *Südstern*.

- From Hotel Steglitz International and from Novum Hotel Ravenna Berlin Steglitz: take U9 until Berliner Straße, then change to U7 in direction Rudow and get off at Südstern (26-30 minutes).
- From Ibis Hotel Berlin City West: take U7 from Fehrbelliner Platz in direction Rudow and get off at Südstern (20 minutes).
- From the Zuse Institute Berlin (ZIB): walk to the U-Bahn station Dahlem-Dorf, take U3 in direction Warschauer Straße until Fehrbelliner Platz, then change to U7 in direction Rudow and get off at Südstern (40 minutes).

Internet Access

At FU Berlin, *eduroam* can be used to access the internet.
A Wi-Fi network will be set up specially for the workshop and can be accessed via a general password. Information about the network name and password will be available at the registration desk at ZIB.

Attendance certificate

If you need an attendance certificate, please send an e-mail to Jean-Philippe Labbé at labbe@zedat.fu-berlin.de.