December 8, 2016

Étale Cohomology

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Exercise sheet 8[1]

Exercise 1. In the course we have shown that any Galois category is equivalent to the category to $\Pi - \mathrm{Fsets}$ with the forgetful functor, where Π is a profinite group and $\Pi - \mathrm{Fsets}$ is the category of finite sets equipped with a continuous Π-action. Use what we do in the class to show that the topological group Π is unique, i.e. if the Galois category is equivalent to $\Pi' - \mathrm{Fsets}$ with the forgetful functor, then we have $\Pi \cong \Pi'$ as topological groups. (Hint: Π was given by the automorphisms of the fiber functor, and the profinite topology is given by a pro-object in the pro-category.)

Exercise 2. We call a locally Noetherian connected scheme X to be simply connected if its étale fundamental group is trivial. Show that if X is simply connected then all finite étale covers of X are of the form $\coprod_{1\leq i\leq n} X_i$ where $X_i = X$.

Exercise 3. Let $(\mathcal{C}, F)$ be a Galois category. Show that the fiber functor F is always faithful.

[1]If you want your solutions to be corrected, please hand them in just before the lecture on December 14, 2016. If you have any questions concerning these exercises you can contact Shane Kelly via shanekelly64@gmail.com or Lei Zhang via l.zhang@fu-berlin.de.