November 2, 2016

Étale Cohomology

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Exercise sheet 3[1]

Exercise 1. Let A be a commutative ring. Let $S \subseteq A$ be a multiplicative subset. Show that the localizazion map $A \to S^{-1}A$ is faithfully flat if and only if it is an isomorphism.

Exercise 2. Let $f : X \to Y$ be a morphism of schemes. The map f is called quasi-compact if there is a covering $\{U_i\}_{i \in I}$ of Y, where U_i are open affine subsets of Y, such that $f^{-1}(U_i)$ is quasi-compact. Show that f is quasi-compact if and only if for any open affine subset $U \subseteq Y$, $f^{-1}(U)$ is quasi-compact.

Remark 1. We have defined the notion of being *of finite type* for a morphism of schemes in Exercise 1.4. There, if we don't assume that the index set for $\{V_j\}$ to be finite, we would get the notion of being *locally of finite type*. Thus we can see clearly from the definition that a morphism is of finite type if and only if it is locally of finite type and quasi-compact.

Exercise 3. Let $f : X \to Y$ be a morphism of schemes which is locally of finite presenation. We say that f is unramified if the diagonal map $\Delta : X \to X \times_Y X$ is an open embedding. The map f is called étale if it is unramified and flat.

Now suppose we have the following commutative diagram

$$\begin{tikzcd} X \arrow[rr, "f"] \arrow[dr, "g"'] & & Y \arrow[dl, "h"] \\ & S & \end{tikzcd}$$

where all the maps are locally of finite presentation. Show the following.

(1) If g is unramified then f is also unramified;
(2) If h is unramified and g is étale then f is étale.

[1]If you want your solutions to be corrected, please hand them in just before the lecture on November 9, 2016. If you have any questions concerning these exercises you can contact Shane Kelly via shanekelly64@gmail.com or Lei Zhang via l.zhang@fu-berlin.de.