February 1, 2017

Étale Cohomology

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Exercise sheet 14[1]

Definition 1. *If* $\mathbf{K}$ *is a triangulated category, and* $\mathcal{A}$ *is an abelian category. An additive functor* $H : \mathbf{K} \to \mathcal{A}$ *is called a cohomological functor if whenever* (A, B, C, u, v, w) *is a distinguished triangle the long sequence*

$$\cdots \xrightarrow{w^*} H(T^i A) \xrightarrow{u^*} H(T^i B) \xrightarrow{u^*} H(T^i C) \xrightarrow{u^*} H(T^{i+1} A) \xrightarrow{u^*} \cdots$$

is exact in $\mathcal{A}$.

Exercise 1. Show that $\mathrm{Hom}_{\mathbf{K}}(X, -)$ is a cohomological functor from $\mathbf{K}$ to the category of abelian groups.

Exercise 2. If (f, g, h) is a morphism of exact triangles in $\mathbf{K}$, and if f, g are isomorphisms, then h is an isomorphism.

$$\begin{tikzcd} X \arrow[r, "u"] \arrow[d, "f"] & Y \arrow[r, "v"] \arrow[d, "g"] & Z \arrow[r, "w"] \arrow[d, "h"] & T(X) \arrow[d, "T(f)"] \\ X' \arrow[r, "u'"] & Y' \arrow[r, "v'"] & Z' \arrow[r, "w'"] & T(X') \end{tikzcd}$$

Exercise 3. If $\mathbf{K}$ is a triangulated category, and if $f : X \to Y$ is a morphism, then by there exist $g : Y \to Z$ and $h : Z \to T(X)$ such that (X, Y, Z, f, g, h) is a distinguished triangle. Show that when f is fixed, (X, Y, Z, f, g, h) unique up to a possibly non-unique isomorphism.

[1] If you want your solutions to be corrected, please hand them in just before the lecture on February 8, 2017. If you have any questions concerning these exercises you can contact Shane Kelly via shanekelly64@gmail.com or Lei Zhang via l.zhang@fu-berlin.de.