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Étale Cohomology

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Exercise sheet 12[1]

Exercise 1. Let X be a connected locally Noetherian scheme, and let $x : \mathrm{Spec}\,(\bar{k}) \to X$ be a geometric point.

(1) Show that $H^1(X_{\text{ét}}, G)$ is equal to the isomorphism classes of G-torsors in the étale topology, where G is a commutative group scheme over X.

(2) Show that for any finite group G the set of continuous maps $\mathrm{Hom}_{\mathrm{cont}}(\pi^1_{\text{ét}}(X, x), G)$ is equal to the isomorphism classes of G-torsors in the étale topology.

(3) Show that $H^1(X_{\text{ét}}, G) \cong \mathrm{Hom}_{\mathrm{cont}}(\pi^1_{\text{ét}}(X, x), G)$ for any finite commutative group G.

(4) What is $H^1(X_{\text{ét}}, \mathbb{Z})$?

Exercise 2. (1) Let X be a proper geometrically reduced and geometrically connected scheme over a perfect field. Show that $H^1(X, \mu_n) = \{\mathcal{L} \in \mathrm{Pic}(X) | \mathcal{L}^{\otimes n} = \mathcal{O}_X\}$.

(2) Let $X = \mathrm{Spec}\,(A)$ be an affine scheme over a field k of characteristic $p > 0$. Show that $H^1(X, \mathbb{Z}/p\mathbb{Z}) = A/\{x^p - x | x \in A\}$.

[1] If you want your solutions to be corrected, please hand them in just before the lecture on February 1, 2017. If you have any questions concerning these exercises you can contact Shane Kelly via shanekelly64@gmail.com or Lei Zhang via l.zhang@fu-berlin.de.