January 18, 2018

Number Theory I (Commutative Algebra)

Hélène Esnault, Exercises: Shane Kelly

Material covered in the course

All references are to the 2017 version of Altman, Kleiman's book.

Week 1. (17.Oct) Chapter 1, Chapter 2 $\leq$ Prop.2.8.
Note: Definition of prime ideal is different from book. "Proper ideal I such that R/I is a domain."

Week 2. (24.Oct) Def.2.10. Thm.2.20. Chapter 3 Radicals (shortened). Chapter 4 Modules (greatly shortened). Chapter 5 Exact sequences (only def, and proj.mod), Thm.5.16.
Exercise sheet: Thm.2.20, PID $\Rightarrow$ UFD.

No class on 31.Oct.

Week 3. (7.Nov) Prop.3.9, Thm.3.14. Lem.3.12. Defn of modules and exact sequences. Scheinnullstellensatz. (Sections 4/5) (4.10): free modules (5.14): projective modules. Thm. (5.16). Lem. (5.17). (8.2).
Exercise sheet: Basis of tensor products and alternating products, and rank of a free module is well-defined.

Week 4. (14.Nov) 8.(1.2.3.4.5.6.8.9.10.16). Flatness finishing with Lazard's theorem. Notes for the second half online.

Week 5. (21.Nov) 11.(1.2.3.8.9.10.11.12.14) 11.29 (exercise) 12.(1.2.3) 12.10 (by hand, no Watts thm), 12.13, 12.16

Week 6. (28.Nov) 14.(1.2.3.4.5.6.7), 10.2, 10.18, 10.20
Exercise sheet: Nakayama's Lemma

Week 7. (5.Dec) 14.4 (2) (part (1) was done previously) (14.5) (14.6) which is very difficult. (14.7) (23.1) (23.2) Ex. $A \subset \hat{A}$ if $A/\mathfrak{m} \subset A$. *Nothing* on depth. (23.6) equivalences ($1 \Leftrightarrow 2 \Leftrightarrow 4$)
Exercise sheet: 14.8

Week 8. (12.Dec) 16.(1.3.5.7) 16.9 (statement), 16.10. Valuation rings (notes on course web page). Thm.23.6($1 \Leftrightarrow 4$ only. Proof given).

Week 9. (19.Dec) Lem.16.9 proof. 15.1 without proof. All of 15.4, 15.7.

No class on 26.Dec. or 2nd.Jan.

Week 10. (9.Jan) Proof of 15.1. Def.24.1, Example.24.2.,Prop.24.6 one direction by hand (i.e., not using Serre's criterion).

Week 11. (16.Jan) 24.(1.2.4.5), 24.6 (using nonbook proof, see notes on course web page), 25.(1.2.3), 25.4 (proof omitted), 25.(5.6.7.8.10.11.12.13.15.16.17).

24.8/25.14 (using nonbook proof, see notes on course web page), 25.(18.19.20.21.22), 24.(10.11.12.13.14.15.16). 25.20 (statement only), 25.22 finishing with the exact sequence

$$1 \to R^* \to K^* \to \mathfrak{F}(R) \to Pic(R) \to 1$$

Week 12. (23.Jan)

Week 13. (30.Jan)

Week 14. (6.Feb)

Week 15. (13.Feb)