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AG Mathematical Geometry Processing

Differential Geometry I
Winter Semester 2025/2026
Freie Universität Berlin

Exercise Sheet 11

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Submission: 19.1.2026, 8:30 AM (start of the tutorial) or 10:15 AM (start of the lecture)

Exercise 1. (7 points)

Consider the associated family of minimal surfaces for the catenoid

$$f(u, v; \theta) := \operatorname{Re} \int_{0}^{z=u+\mathrm{i}v} \mathrm{e}^{\mathrm{i}\theta} \varphi(\xi) \, \mathrm{d}\xi + c \tag{1}$$

for $u, v, \theta \in \mathbb{R}$ and $c \in \mathbb{R}^3$ with the function

$$\varphi(z) := \begin{pmatrix} \sinh(z) \\ \mathrm{i}\cosh(z) \\ 1 \end{pmatrix} . \tag{2}$$

i) Prove, that this reproces the family of surfaces from Exercise 1 on Sheet 4, after applying a suitable transform to the parameters u, v, θ and a suitable choice of the integration constant c.

ii) Then, solve Parts i) to iii) Exercise 1 on Sheet 4 again, using your knowledge about the Weierstraß-Enneper presentation.

iii) Sketch both the catenoid and the helicoid, and illustrate the parameter lines that correspond to each other.

Exercise 2. (5 points)

Consider the associated family of the Enneper surface given by Equation (1), but with $\varphi(z)$ obtained from the functions $F(z) = z$ and $G(z) = 1$ in the Weierstraß-Enneper presentation. Prove the relation

$$f\left(\mathrm{e}^{\mathrm{i}\theta/2} \cdot z; \theta\right) = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) & 0 \\ \sin(\theta/2) & \cos(\theta/2) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot f(z; 0) \tag{3}$$

and explain why this shows, that the associate family $f(-, -; \theta)$ of the Enneper surface is just the Enneper surface rotating by an angle $\theta/2$ around the z-axis.

Exercise 3. (4 points)

Let $n \in \mathbb{N}$ with $n \geq 3$. Consider n triangles in Euclidean space connected to each other in a circular fashion around a single common vertex p.

i) What is the range of values, that the discrete Gauss curvature (as defined by the angle defect) at the point p can have, when the other vertices of the triangles can be freely arranged and their number n is allowed to vary freely.

ii) Answer Part i) again, but for any fixed number of triangles n.