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AG Mathematical Geometry Processing

Differential Geometry I
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Freie Universität Berlin

Exercise Sheet 2

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Submission: 3.11.2025, 8:30 AM (start of the tutorial) or 10:15 AM (start of the lecture)

Exercise 1. (8 points)

The map

$$\gamma : \left[0, \frac{2\pi}{\alpha}\right] \to \mathbb{R}^3, t \mapsto \begin{pmatrix} r\cos(\alpha t) \\ r\sin(\alpha t) \\ ht \end{pmatrix} \tag{1}$$

parametrizes a so called *helix* with radius $r > 0$, $\alpha \in \mathbb{R} \setminus \{0\}$, and slope $h \in \mathbb{R}$.

i) Determine $L\left(\gamma|_{[0, \frac{2\pi}{\alpha}]}\right)$.

ii) Find a parametrization by arc length of γ.

iii) Let $\alpha = 1$. Determine all r and h such that γ is parametrized by arc length already. Sketch your results.

iv) Determine the curvature and torsion of γ.

Exercise 2. (4 points)

Let $\gamma : \mathbb{R} \to \mathbb{R}^2$ be a regular C^2 curve. Show that if all tangents of γ intersect in one point then γ is a straight line.

Exercise 3. (4 points)

Let $\gamma : I \to \mathbb{R}^3$ be a Frenet curve with Frenet frame $\{\gamma'(s), n(s), b(s)\}$. For a smooth function φ the one-parameter family of rotations

$$\begin{pmatrix} \tilde{n}(s) \\ \tilde{b}(s) \end{pmatrix} = \begin{pmatrix} \cos\varphi(s) & -\sin\varphi(s) \\ \sin\varphi(s) & \cos\varphi(s) \end{pmatrix} \begin{pmatrix} n(s) \\ b(s) \end{pmatrix} \tag{2}$$

generates a new orthonormal frame $\left\{\gamma'(s), \tilde{n}(s), \tilde{b}(s)\right\}$.

i) Compute the torsion $\tilde{\tau}(s) = \left\langle \tilde{n}'(s), \tilde{b}(s) \right\rangle$ of the new frame.

ii) Find a function φ such that the new frame is torsion free, i.e. $\tilde{\tau}(s) = 0$ for all s. Such a frame is called a *parallel frame*.

iii) Find a parallel frame for the arc length parametrized helix from exercise 1 on this sheet.