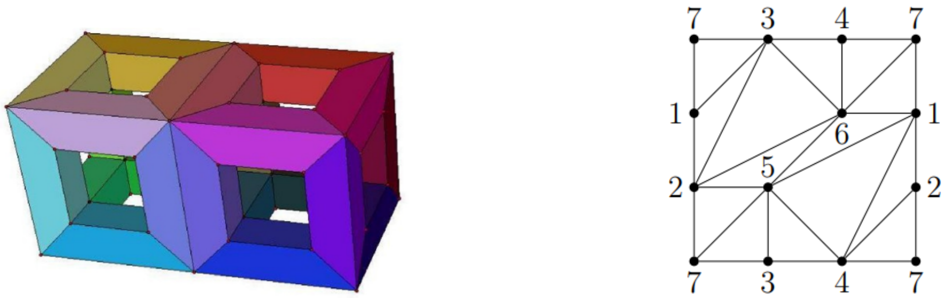
Prof. Dr. Konrad Polthier
Eric Zimmermann
Version: 1

Differential Geometry I
Winter Semester 2023/2024
Freie Universität Berlin

Exercise Sheet 11

Submission: 06.02.2024, 12:15 PM (start of lecture)

Note: This sheet contains 2 bonus points.



(a) (b)

Figure 1: Left: Quadrilateral surface. Right: Drawing of *Császár torus*.

Exercise 1. (1+3+2+2+2 = 10 points)

i) Determine the genus of a closed simplicial surface with 20 triangles and 12 vertices.

ii) Sketch[1] two nonisomorphic examples of a closed simplicial surface with 20 triangles and 12 vertices which are both not the icosahedron. Why are your examples nonisomorphic?

iii) How many edges does a simplicial double torus with 1200 vertices have?

iv) Let Q be a quadrilateral surface (see Figure 1a for an example) and v, e, f, g, and χ denote the number of vertices, number of edges, number of faces, genus, and Euler characteristic of Q. Determine v, e, f, g, and χ for the example shown in Figure 1a.

v) Give three examples for simplicial complexes having the same Euler characteristic but which are not simplicially isomorphic[2]. Justify your choice.

Exercise 2. (3 points)
The *Császár torus*[3] is a two-dimensional simplicial surface consisting of 7 vertices, 21 edges, and 14 triangles, cf. Figure 1b. Vertices labeled with the same index are identified and the edges are identified accordingly. Determine

- star([2]), star([3,4]), star([1,5,6]),
- link([2]), link([3,4]), and link([1,5,6]).

Please use the definition of a star stated in the script.

Exercise 3. (2+3 = 5 points)

i) Show that every m-simplex has 2^{m+1} faces, i.e. subsimplices for $k \in \{0, \ldots, m\}$.

ii) The Euler characteristic can be translated to higher dimensions via defining

$$\chi(K) = \sum_{k \geq 0} (-1)^k f_k(K)$$

where f_k denotes the number of k-faces of a simplicial complex K. Let Δ^n denote the n-simplex. Show $\chi(\Delta^n) = 1$.

[1] Think of an appropriate representation.
[2] They are not isomorphic as simplicial complexes.
[3] A model of the *Császár torus* can be found in JavaView, i.e. in File - Open - JavaView Models in the category Polytope.