

## Differential Geometry II – Homework 10

Submission: July 08, 2022, 10:15 am

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### 1. Exercise (5 points)

Show that the set of *deck transformations*  $\text{deck}$  of a covering space  $E$  forms a group under composition of functions. Is this group commutative? Justify your solution.

### 2. Exercise (3 points)

Let  $\Gamma$  denote the group of diffeomorphisms of a manifold  $M$  and  $M/\Gamma$  its quotient space consisting of equivalence classes  $[p]$  where

$$p \sim q \Leftrightarrow \exists \Phi \in \Gamma : q = \Phi(p).$$

Show that  $[p]$  is indeed an equivalence class.

### 3. Exercise (8 points)

Show that  $\pi : \mathbb{S}^1 \rightarrow \mathbb{S}^1, z \mapsto z^n, n \in \mathbb{N}$ , is a covering of the unit circle (here, in the complex plane). Further, show that  $\delta : \mathbb{S}^1 \rightarrow \mathbb{S}^1, z \mapsto ze^{\frac{2\pi i}{n}}$ , is a deck transformation and that  $\text{deck}(\pi) \cong \mathbb{Z}/n\mathbb{Z}$ .

Total: 16