

## Differential Geometry II – Homework 09

Submission: July 01, 2022, 10:15 am

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### 1. Exercise (5 points)

Let  $M$  be a manifold,  $p \in M$  a point and denote by  $\pi_1(M, p)$  the first fundamental group of  $M$  at  $p$ . Give examples for  $M$  and  $p$  such that

- 1.)  $\pi_1(M, p)$  is Abelian,
- 2.)  $\pi_1(M, p)$  is not Abelian.

Justify your solutions.

### 2. Exercise (5 points)

By considering the covering  $\mathbb{S}^2 \rightarrow \mathbb{R}P^2$ , prove that  $\pi_1(\mathbb{R}P^2)$  contains a non-contractible loop. Show further that any non-contractible loop in  $\mathbb{R}P^2$  is null-homotopic in  $\mathbb{R}P^2$  if it is passed through twice. Illustrate your arguments by providing a sequence of sketches of the homotopy.

### 3. Exercise (6 points)

Find a Möbius strip on the non-orientable Klein bottle. Visualize your solution.

Total: 16