Freie Universität Berlin SoSe 22
Institut für Mathematik
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Differential Geometry II – Homework 08

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1. Exercise (3 points)
Let (M, g) be a Riemannian manifold with a unit-speed geodesic $\gamma : I \subset \mathbb{R} \to M$. A Jacobi field J along γ with $J \perp \gamma'$ everywhere is called a *normal* Jacobi field along γ. Show that if (M,g) has constant sectional curvature $C \in \mathbb{R}$, then for normal Jacobi fields the Jacobi equation simplifies to $J'' + CJ = 0$.

2. Exercise (5 points)
Consider the upper half plane $\mathbb{H} := \{(x, y) \in \mathbb{R}^2 | y > 0\}$ equipped with the metric

$$g_p := \frac{1}{y^q}\delta_{ij}$$

where $q \in \mathbb{R}_+$ and $q \neq 2$. Show that $(\mathbb{H}, g_q)$ is not geodesically complete.

3. Exercise (8 points)
Let M and $\tilde{M}$ be two geodesically complete, connected Riemannian manifolds and let $\pi : \tilde{M} \to M$ be a *local isometry*, i.e. for each point $p \in \tilde{M}$, there exists an open neighborhood $U \subseteq \tilde{M}$ of p such that $\pi|_U$ is an isometry.

1.) Show that π fulfils the *lifting property for geodesics*: for every geodesic $\gamma : [0, 1] \to M$ and each point $p \in \tilde{M}$ with $\pi(p) = \gamma(0)$ there exists a unique geodesic $\tilde{\gamma} : [0, 1] \to \tilde{M}$ such that $\pi(\tilde{\gamma}) = \gamma$ and $\tilde{\gamma}(0) = p$.

2.) Show that π is surjective.

3.) Conclude that π is a smooth covering map, i.e., is smooth surjective map with the property that every $p \in M$ has a connected neighborhood V such that each component of $\pi^{-1}(V)$ is mapped diffeomorphically onto V by π.

Total: 16