Freie Universität Berlin SoSe 22
Institut für Mathematik
Prof. Dr. K. Polthier, H.-S. Lipschütz Status: June 3, 2022

Differential Geometry II – Homework 07

Submission: June 17, 2022, 10:15 am

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1. Exercise (4 points)
Consider the manifold $U = \{p \in \mathbb{R}^n \mid \|p\| < 1\}$ equipped with the hyperbolic metric

$$g_{ij}|_p := \frac{4}{(1 - |p|^2)^2}\delta_{ij}.$$

Determine

1.) the Ricci curvature Ric,

2.) and the scalar curvature S

of (U, g) explicitly.

2. Exercise (4 points)
Let (M, g) be a Riemannian manifold and let $\{E_1, \ldots, E_{n-1}, X\}$ be an orthonormal basis of T_pM at some point p in M. Denote with $(\mathbb{T}_p)_i$ the plane in T_pM spanned by E_i and X. Show that

$$\mathrm{Ric}(X, X) = \sum_{i=1}^{n-1} K_{(\mathbb{T}_p)_i}$$

where $K_{(\mathbb{T}_p)_i}$ denotes the sectional curvature.

3. Exercise (4 points)
Consider the upper half plane $\{(x, y) \in \mathbb{R}\times]0, \infty[\}$ equipped with the metric

$$g = \frac{1}{y^2}(\delta_{ij}).$$

Let γ be a parameterization of constant speed of $\{(x_0, y) | y \in]0, \infty[\}$ for fixed x_0.

1.) Show that $J = \partial_x$ is a Jacobi field along γ.

2.) Knowing the geodesics in the upper half plane, find a non-tangential Jacobi field. Justify your solution.

Please turn over.

4. Exercise (4 points)

Let (M, g) be a Riemannian manifold with normal coordinates (U, x_i) around $p \in M$ (see exercise sheet 04). For $W = \sum_{i=1}^n W_i \partial_i \in T_pM$, show that the Jacobi field along a radial geodesic γ (i.e. $\gamma(0) = p$) with $J(0) = 0$ and $J'(0) = W$ is given by $J(t) = t \sum_{i=1}^n W_i \partial_i$ for all t.

Total: 16