Freie Universität Berlin
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SoSe 22

Status: May 31, 2022

Differential Geometry II – Homework 06

Submission: June 10, 2022, 10:15 am

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1. Exercise (4 points)

Let M be a Riemannian manifold with constant sectional curvature $K_{\mathbb{T}_p} = C$. Show that $R(V,W)X = C\left(\langle X,W\rangle V - \langle X,V\rangle W\right)$ holds.

2. Exercise (4 points)

Let M be a two-dimensional Riemannian manifold. Show that the sectional curvature $K_{\mathbb{T}_p}$ in $p \in M$ coincides with the Gaussian curvature in p explicitly.

3. Exercise (4 points)

Let (M,g) be a Riemannian manifold and $\overline{M} \subset M$ a submanifold of codimension 1 with normal field N. You may use without proof that $\overline{\nabla}_V W = \nabla_V W + b(V,W)N$ where $b(V,W) := \langle \nabla_V N, W\rangle = -\langle \nabla_V W, N\rangle$.

1.) Show that $\langle \overline{R}(V,W)X,Y\rangle = \langle R(V,W)X,Y\rangle + b(V,Y)b(W,X) - b(V,X)b(W,Y)$.

2.) Deduce that for a plane $\Pi_p \subset T_p\overline{M} \subset T_pM$ it is $\overline{K}(\Pi_p) = K(\Pi_p) + \det b\,|_{\Pi_p}$.

4. Exercise (4 points)

Consider the manifold $U = \{p \in \mathbb{R}^n \mid \|p\| < 1\}$ equipped with the hyperbolic metric

$$g_{ij}|_p := \frac{4}{(1-|p|^2)^2}\delta_{ij}.$$

Determine

1.) the curvature tensor R,

2.) the sectional curvature K

of (U,g) explicitly.

Total: 16