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Complex Analysis – Homework 10

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1. Exercise (2 points)
Give an example of a region $\Omega \subset \mathbb{C}$ and a holomorphic function $f : \Omega \to \mathbb{C}$ without zeros that does not have a holomorphic logarithm ("$\log(f) = g$"). Justify your solution.

2. Exercise (6 points)
For $k \in \{1, 2, 3, 4\}$, let $\Omega_k \subset \mathbb{C}$ be the domain of definition and $D_k \subset \Omega_k$ a discrete subset consisting of the singularities of $f_k : \Omega_k \setminus D_k \to \mathbb{C}$. Determine D_k for the following functions f_k and classify their singularities as removable, pole, or essential:

1.) $f_1(z) = \frac{z+2}{z^2+2z+5}$,

2.) $f_2(z) = \frac{1}{z^2(1-\exp(2\pi iz))}$,

3.) $f_3(z) = \frac{\cos(z)-1}{z^6}$,

4.) $f_4(z) = \frac{1}{\sin(z)-\cos(z)}$.

3. Exercise (4 points)
Let $\mathbb{D}^* := \{z \in \mathbb{C} \mid |z| < 1\} \setminus \{0\}$ and $f : \mathbb{D}^* \to \mathbb{C}$ a holomorphic function fulfilling $|f(z)| \leq M|z|^t$ for some constant $M > 0$, $t > -1$, and all $z \in \mathbb{D}^*$. Show that f has a removable singularity at $z = 0$.

4. Exercise (4 points)
Let $\Omega \subset \mathbb{C}$ be a domain and $f : \Omega \to \mathbb{C}$ a holomorphic function. Show: If $z_0 \in \Omega$ is a pole or an essential singularity of f, then z_0 is not a pole of $\exp(f)$.

Total: 16