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Complex Analysis – Homework 9

Submission: June, 22th, 2021, 10:15 am, via email

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1. Exercise (4 points)
For the principal branch, determine the logarithm of the following numbers $z_i \in \mathbb{C}^*$, $i \in \{1,2,3,4\}$:

1.) $z_1 = -i$,

2.) $z_2 = 1 + i$,

3.) $z_3 = (i(i-1))^i$,

4.) $z_4 = i^i(i-1)^i$.

2. Exercise (4 points)
Let $z, w \in \mathbb{C}^*$. Show the following identity for the complex logarithm on its principal branch:

$$\log(zw) = \log(z) + \log(w) + 2\pi i k(z,w),$$

where

$$k(z,w) := \begin{cases} 0 & \text{if } -\pi < \arg(z) + \arg(w) \leq \pi, \\ 1 & \text{if } -2\pi < \arg(z) + \arg(w) \leq -\pi, \\ -1 & \text{if } \pi < \arg(z) + \arg(w) \leq 2\pi. \end{cases}$$

3. Exercise (4 points)
Show the following statements:

1.) There is no continuous function $f : \mathbb{C}^* \to \mathbb{C}^*$ such that

$$f(zw) = f(z)f(w) \text{ and } (f(z))^2 = z \text{ for all } z, w \in \mathbb{C}^*.$$

2.) There is no continuous function $g : \mathbb{C}^* \to \mathbb{C}$ such that $\exp(g(z)) = z$ for all $z \in \mathbb{C}^*$.

Please, turn over.

4. Exercise (4 points)

Let $w, \tilde{w} \in \mathbb{C}^*$ be two complex numbers which are linearly independent over $\mathbb{R}$. Show that every entire function $f : \mathbb{C} \to \mathbb{C}$ fulfilling

$$f(z + w) = f(z) = f(z + \tilde{w}) \text{ for all } z \in \mathbb{C}$$

is constant.

Total: 16