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Complex Analysis – Homework 8

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1. Exercise (4 points)

Let $\mathbb{D} := \{z \in \mathbb{C} \mid |z| < 1\}$. Decide whether there is a holomorphic function $f_k : \mathbb{D} \to \mathbb{C}$, $k \in \{1, 2, 3, 4\}$, fulfilling the given properties:

1.) $f_1\left(\frac{1}{n}\right) = f_1\left(-\frac{1}{n}\right) = \frac{1}{n^2}$ for all $n > 1$,

2.) $f_2\left(\frac{1}{n}\right) = (-1)^n \frac{1}{n}$ for all $n > 1$,

3.) $f_3\left(\frac{1}{2n}\right) = f_3\left(\frac{1}{2n-1}\right) = \frac{1}{n}$ for all $n > 1$,

4.) $f_4^{(n)}(0) = (n!)^2$ for all $n \geq 0$.

2. Exercise (4 points)

For the following functions f_1 and f_2, determine their maximal domain of definition, their Taylor series centered at $z_0 \in \mathbb{C}$, and their radii of convergence:

1.) $f_1(z) = \frac{1}{z}$, $z_0 = 1$,

2.) $f_2(z) = \frac{1}{z^2 - 5z + 6}$, $z_0 = 0$.

3. Exercise (4 points)

Show the following statement: Let $\Omega \subseteq \mathbb{C}$ be a domain and $f : \Omega \to \mathbb{C}$ a non-constant holomorphic function. Then $f(\Omega)$ is a domain, too.

4. Exercise (4 points)

Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function and $R, M \in \mathbb{R}_{>0}$ constants. Further, there exists $n \in \mathbb{N}$ such that $|f(z)| \leq M|z|^n$ for all $z \in \mathbb{C}$ with $|z| \geq R$. Show that f is a polynomial of degree $\leq n$.

Total: 16