Freie Universität Berlin SoSe 21
Institut für Mathematik
Prof. Dr. K. Polthier, H.-S. Lipschütz Status: 10. Mai 2021

Complex Analysis – Homework 4

Submission: May, 18th, 2021, 10:15 am, via email

---

1. Exercise (5 points)
Consider the curve $\gamma : [0, 4\pi] \to \mathbb{C}$ fulfilling $\gamma(0) = \gamma(4) = 1$, $\gamma(1) = i$, $\gamma(2) = -1$, and $\gamma(3) = -i$. Additionally, γ is affine linear on each interval $[k, k+1]$, $k \in \{0, \dots, 3\}$.

1.) Parameterize γ explicitly.

2.) Determine the following integral directly (i.e. without using path independence or Cauchy's theorem):

$$\frac{1}{2\pi i} \int_\gamma \frac{1}{z} \, dz.$$

2. Exercise (5 points)
Let $G \subset \mathbb{C}$ be an open convex set and let $f : G \to \mathbb{C}$ be holomorphic with continuous derivative f'. Assume that

$$|f'(z) - 1| < 1 \text{ for all } z \in G.$$

The inequality assures theat f' is close to 1. Show that f is injective on G.

3. Exercise (6 points)
Show the following (weakened) version of the theorem of Goursat:
Let $\triangle$ be a closed triangle in $\mathbb{C}$ and $z_0 \in \triangle$. Further, let $U \subset \mathbb{C}$ be an open neighborhood of $\triangle$ and $f : U \to \mathbb{C}$ be a continuous function which is holomorphic on $U \setminus \{z_0\}$. Then, the following holds:

$$\int_{\partial\triangle} f(z) \, dz = 0.$$

Total: 16